Deliverable D2.1

A Survey Report on Applications of Combinatorial Optimization Methods in Urban Planning and Relevant Mathematical Models

COVER

Combinatorial **O**ptimization
for **V**ersatile Applications to **E**merging u**R**ban Problems

The Marie Skłodowska-Curie Grant Agreement No: 101182819

This work has been supported by
the European Commission's Horizon Europe Research and Innovation programme
through the Marie Skłodowska-Curie Actions Staff Exchanges (MSCA-SE)
under Grant Agreement no. 101182819 (COVER: (C)ombinatorial (O)ptimization
for (V)ersatile Applications to (E)merging u(R)ban Problems)

Contents

Introduction

In mathematics, the art of proposing a question must be held of higher value than solving it.
Georg CANTOR

Urban planning in the 21st century faces a wide range of challenges, including the development of efficient transportation systems, the pursuit of environmental sustainability, the enhancement of emergency response, and equitable access to public services. These challenges are intensified by factors such as rapid urban population growth and climate change. As cities become increasingly complex, the analytical and planning tools used to understand and improve them must evolve accordingly.

Combinatorial optimization has emerged as a powerful mathematical framework for modeling, analyzing, and solving many of the complex challenges inherent in modern urban development. These methods are especially well suited to urban problems that can be represented as networks or graphs, where resources must be allocated, flows optimized, or connections established under intricate and often competing constraints. Complementary branches of combinatorial optimization, such as integer programming, structural graph theory, operations research, and network optimization, each contribute distinct perspectives to urban challenges. When integrated, these techniques allow researchers and practitioners to capture the structural, spatial, and dynamic complexities of urban systems. This holistic approach not only provides rigorous theoretical foundations, but also equips decision-makers with practical tools to address critical domains such as transportation, energy, healthcare, disaster management, urban security, and accessibility to services.

The purpose of this report is to present a structured survey on the applications of combinatorial optimization in urban planning, with particular emphasis on the underlying mathematical models. Each chapter highlights a specific domain in which these methods have proven not only beneficial but often essential to develop efficient and sustainable solutions. Beyond reviewing existing work, the report also seeks to identify methodological trends, key challenges, and promising directions for future research.

ORGANIZATION OF THE REPORT. The report is organized into ten chapters, each addressing a distinct yet complementary area of urban planning where combinatorial optimization and graph-theoretic methods play a decisive role.

- **Chapter 1: Facility Location** discusses facility location models for emerging micromobility services, such as e-scooters and shared bicycles, with a focus on allocating appropriate parking infrastructure under docked and dockless systems. It also examines relevant graph-theoretic models that provide a structured framework for addressing these challenges.

- **Chapter 2: Bike-Sharing Systems** surveys the static bike rebalancing problem (SBRP), emphasizing problem variants, mathematical models, and solution approaches aimed at addressing supply-demand imbalances in bike-sharing systems.
- **Chapter 3: Charging Systems** reviews the operational challenges and models in e-scooter charging systems, reviewing the combinatorial optimization models for routing and battery-swapping operations.
- **Chapter 4: Public Transportation** examines optimization problems across the strategic, tactical, and operational levels of public transportation planning, including line planning, timetabling, vehicle and crew scheduling, and integrated solution methods.
- **Chapter 5: Disaster Management** highlights applications of combinatorial optimization in pre-disaster planning, shelter allocation, humanitarian logistics, and evacuation strategies, with particular attention to uncertainty, resilience, and equity.
- **Chapter 6: The 15-minute City** investigates computational and mathematical approaches to the 15-minute city paradigm, including grid tessellation, accessibility analysis, and optimization methods that promote sustainable and equitable urban design.
- **Chapter 7: Urban Street Cleaning** explores urban street cleaning as a practical application of arc routing problems, extending the discussion to related contexts such as snow removal, waste collection, and digital mapping.
- **Chapter 8: Mobile Clinics** presents the vehicle routing problem (VRP) in the context of mobile clinics in rural South Africa, showing how optimization supports equitable access to healthcare under logistical and service constraints.
- **Chapter 9: Patrolling Streets** presents the Armed Response problem as a resource allocation model within urban security systems, examining its connections to police patrol and emergency response models and situating the problem within graph-theoretic frameworks such as domination and network flow models.
- **Chapter 10: Electric Power Systems** surveys graph-theoretic and optimization approaches to planning and operating power systems, with applications in clustering, reliability assessment, generation scheduling, and renewable energy integration.

Collectively, these chapters highlight the extensive scope and profound impact of combinatorial optimization in addressing complex and pressing urban challenges, demonstrating how advanced mathematical and graph-theoretic methods can inform efficient, resilient, and sustainable solutions across diverse aspects of city planning and management. By providing both conceptual overviews and application-driven insights, this report seeks to serve as a resource for researchers, practitioners, and policymakers who aim to design more efficient, resilient, and sustainable cities.

Chapter I

Facility Location

The rapid growth of micromobility services, such as e-scooters and shared bicycles, has introduced new challenges in urban infrastructure planning. One of the most pressing issues is the allocation of appropriate parking infrastructure. Known graph theoretic solutions for the facility location problem in both dockless and docked operating systems are discussed here.

Keywords: Facility location, domination, covering problem, clustering, centrality, fuzzy graphs.

1

Facility Location in a Micromobility Environment

Dragana Božović, University of Maribor, Slovenia
José Carlos Camacho Moreno, University of Cadiz, Spain
Tanja Dravec, University of Maribor, Slovenia
Lara Drožđek, University of Maribor, Slovenia
Aleksander Kelenc, University of Maribor, Slovenia
Dorota Kuziak, University of Cadiz, Spain
María José Pereira-Sáez, University of A Coruña, Spain
Iztok Peterin, University of Maribor, Slovenia
Adriana Roux, Stellenbosch University, South Africa
Valisoa Rakotonarivo, University of Pretoria, South Africa
Aleksandra Tepeh, University of Maribor, Slovenia
Juan Carlos Valenzuela Tripodoro, University of Cadiz, Spain
Ismael G. Yero, University of Cadiz, Spain

Public transportation enables efficient travel between key points within urban areas. However, transit stations cannot be located near every residence, giving rise to the "first/last mile" problem. Micromobility solutions, such as shared bicycles and e-scooters, offer a flexible and effective means to address this gap, facilitating convenient access to and from public transport nodes.

Among the first attempts to solve this first/last mile problem was the "White Bicycle Plan" program launched in Amsterdam in 1965. This program left 50 white-painted bicycles around the city, free for all to use. Unfortunately, it failed due to theft and vandalism. The pioneer of modern bike-sharing can be seen as the "Bycyklen" programme from Copenhagen in 1995, where you could deposit 20 Danish Krone to use a bicycle that was refunded when you returned the bicycle [49].

The first studies on micromobility began in the late 2000s and gained more momentum in the following decade when lightweight devices such as electric bicycles and electric scooters (e-scooters) became more popular [19, 21, 49].

The rapid growth of micromobility services, such as e-scooters and shared bicycles, has introduced new challenges in urban infrastructure planning. One of the most pressing issues is the allocation of appropriate parking infrastructure to reduce sidewalk clutter, improve accessibility, and ensure regulatory compliance. When considering a dockless operating system, as in the case of e-scooters, parking locations for the deployment of the scooters (this can either be in the initial deployment or in the deployment of scooters gathered from remote places) should be determined. On the other hand, an operating system that makes use of docking stations (as in the case of some shared bicycle systems), the location of these stations must be optimal.

In this chapter we consider facility location in a micromobility environment, mainly focusing on graph-theoretic approaches to solve this problem. In Section 1.1, we discuss relevant graph-theoretic solutions. We discuss the complexity of the problems in Section 1.2 and we conclude the chapter in Section 1.3.

1.1 Station placement models in micromobility

The station placement problem in micromobility can be formulated using graph-theoretic concepts. Consider a city or geographic region represented as a graph, where vertices correspond to physical locations and edges represent connections or distances between them. The goal is to identify an optimal subset of locations for deploying bicycles or e-scooters, maximizing accessibility, coverage, and overall efficiency of the system. This formulation captures the challenge of selecting facility sites that effectively serve demand while minimizing user travel distances and ensuring efficient resource utilization.

This section presents several known graph-theoretic solutions to the station placement problem in micromobility. We consider the following approaches: covering location problems, clustering methods, centrality measures, domination parameters and fuzzy graphs.

1.1.1 Covering location problems

The facility location problem with given demand is classically solved by the Maximum Covering Location Problem (MCLP) first introduced in [15]. In this seminal paper the MCLP was introduced to determine the placement of a fixed number of facilities that will maximize the demand that can be served within a given service distance. A different variant of this problem where every vertex in the graph outside of the service distance is required to be within a certain allowable distance from the chosen facilities is also discussed. These problems can be solved by modeling them as mixed integer linear programming (MILP) problems with a coverage and facility constraint. The original MCLP was further expanded by Yue et al. [53] through the introduction of a distance tolerance by considering the distance a user is willing to walk before picking up an e-scooter. They posed the problem as an MILP problem and then made use of deep reinforcement learning to solve the problem for the city of Chigaco, USA. In [16] the problem of finding the optimal placement of stations for e-scooters was modeled as a multi-objective Maximum Coverage Parking Location problem maximizing public transportation accessibility, demand, and taking points of interest into account. This study includes a case study on the city of Rome. A maximum demand coverage problem used to allocate bicycle stations is introduced in [35] with a case study on data from a bike sharing system in Santiago, Chile. In [31] a capacitated maximal covering location problem is used to show how the placement of facilities can improve the coverage of the existing stations of the bike-sharing program in Boulder, Colorado.

1.1.2 Clustering algorithms

Data-driven approaches, particularly clustering algorithms, have been widely adopted to identify optimal locations for parking facilities. A variety of clustering techniques have been employed, each with distinct advantages and limitations.

In [7] the K-means algorithm was applied to aggregate trip origins and destinations into 180 candidate station sites. After generating candidate station sites via K-means, a p-median mixed-integer programming model was applied to select the optimal subset of stations. Constraints such as maximum walking distance, proximity to bike lanes, and facility capacity were incorporated. Additionally, queueing theory and dwell-time analysis were also used for station sizing. This integration ensures that facility placement is both data-driven and operationally feasible. Data from a shared e-scooter network in Thessaloniki, Greece is used to test these methods.

Then, in [47], the authors extended clustering approaches by using density-based clustering algorithms (DBSCAN and HDBSCAN). Their results showed that density-based approaches outperform K-means in identifying meaningful clusters of trip endpoints, especially in environments with irregularly shaped or variable-density clusters. Unlike K-means, DBSCAN and HDBSCAN can handle noise and do not require predefining the number of clusters. Nevertheless, these methods rely on hyperparameters such as neighborhood radius and minimum cluster size, which require careful tuning. They further introduce the use of weighted clustering, wherein certain trip endpoints were assigned higher importance based on policy-relevant attributes, such as narrow sidewalks. This weighting improved the ability of clustering algorithms to capture efficiently "problematic" trips, although it led to a modest decline in overall trip capture. This trade-off highlights the potential of clustering as a flexible tool for balancing general demand coverage with equity and accessibility concerns. The city of Nashville, USA is used as a case study.

1.1.3 Centrality measures

In transportation network analysis, centrality measures play a crucial role in guiding facility location decisions, such as the placement of terminals, depots, or transfer hubs. A wide variety of centrality measures has been proposed, and an extensive overview can be found in the Periodic Table of Network Centrality [48]. Here, we highlight only those measures that are particularly relevant for addressing the problem of facility location planning in transportation networks.

Degree centrality identifies nodes with the most direct connections, making them natural candidates for facilities that benefit from high local accessibility [23]. However, locally important nodes are not necessarily globally important; therefore, it is also necessary to consider centrality measures that capture the role of nodes in the broader network structure.

Betweenness centrality highlights nodes that frequently lie on the shortest paths between others [22]. High betweenness centrality values reflect potential bottlenecks in the network, which is particularly valuable when locating facilities at strategic transfer points to minimize congestion and improve overall flow efficiency.

Closeness centrality emphasizes nodes with minimal average distance to all others [46]. This measure supports facility placement decisions aimed at maximizing global accessibility and reducing travel times across the network.

Finally, eigenvector centrality helps identify nodes connected to other influential nodes, offering insight into facility locations that not only serve direct demand but are also embedded within important corridors of movement [10].

Taken together, these measures provide a robust methodological foundation for facility location planning in transportation networks. For example, Bora and Baruah [11] applied degree centrality, betweenness centrality, and closeness centrality to real-world data from the Dispur urban transportation network in India, demonstrating their practical relevance. Agryzkov et al. [1] proposed a centrality model for urban networks based on the concept of eigenvector centrality and applied it to empirical data collected through fieldwork on commercial activity in the city of Murcia, Spain. Another example of a practical application using multiple types of centrality measures was provided by Napitupulu et al. [38], who applied degree centrality, betweenness centrality, closeness centrality, eigenvector centrality, and weighted eigenvector centrality to analyze the transportation network of the Unpad Jatinangor Campus route in Indonesia.

1.1.4 Graph domination parameters

The notion of domination in graph theory arises from the requirement that every vertex in a graph is either in a designated set of vertices or adjacent to one. Basic and also specialized information on domination in graphs can be found in the books [26, 27, 28, 29]. Domination in graph theory can model facility location problems [17, 20, 24, 32, 45]. Zones of a city are represented as vertices, and a dominating set corresponds to choosing the smallest set of zones where facilities are placed so that every zone either hosts a facility itself or is adjacent to one. This captures the idea of efficient allocation with full coverage.

Beyond transportation-specific settings, domination models have also been applied to broader urban resource allocation tasks, such as optimizing the distribution of services using minimum dominating sets in interval graphs [42].

This classical definition of domination underpins a wide range of domination variants (see [26, 27, 28] for several of these variants), each of which has been explored both theoretically and in applications such as resource allocation and facility location [45]. Depending on the specific requirements of location problems, a variety of domination variants have been introduced.

Multiple domination reflects real-world allocation problems where redundancy is desirable. Gagarin and Corcoran introduced models of *multiple domination* to optimize the placement of electric vehicle charging stations in road networks [24]. Here, each demand node is required to be dominated by more than one facility, ensuring users have access to multiple charging options within range. Their study developed integer programming formulations and heuristics, showing that multiple domination is a natural model for resilient facility placement.

The *k-domination* framework has also been explored in depth for road networks. Corcoran and Gagarin proposed heuristics for k-domination models applied to facility location on urban street networks [17]. Their work demonstrated that greedy and beam-search algorithms are effective for finding small k-dominating sets in large graphs, providing scalable methods for street-level allocation. Later, Dijkstra et al. extended this line of research to directed graphs [20] (a graph is directed if the edges are replaced by arrows - the notion of a direction is used). By considering road networks as digraphs, they accounted for directionality in travel (e.g., one-way streets), and proposed greedy heuristics for k-domination under these constraints. Both contributions show how domination models naturally encode the allocation of urban facilities to demand points with redundancy and direction-aware reachability.

There are also many other domination type parameters that are important for micromobility systems. In *distance-r domination*, adjacency is replaced by distance at most r in the graph (the distance between vertices in a graph is understood as the smallest number of edges that can be used to arrive from one vertex to the other one through adjacency). A distance-r dominating set D guarantees that every vertex lies within a distance r to some vertex $u \in D$. This corresponds directly to walking-radius constraints in facility placement [29]. Different values of r model tolerances such as 200m or 400m access distances to docking stations or scooter deployment points.

The question of whether micromobility services are accessible to and equally distributed to all clients in the network is mostly considered when evaluating the current state of the system and not during the design phase. By evaluating the current state of a network Aman et al. [6] showed that disadvantage and low-income communities were less likely to have access to bikes and scooters. In [36], a dockless bicycle sharing system in Seattle was investigated for equity and it was shown that there is a slight shift in availability to higher income communities. Caggiani et al. [12] considered equity as part of the design process and proposed a model that minimizes inequalities but at the same time maintain certain levels of accessibility and coverage. In [8] the equity of the distribution of stations was considered when changing from a dockless to a docked system where they maximize equity while minimizing the total walking distance. A *fair dominating set* D is a dominating set in which every vertex outside D is dominated by the same number k of vertices of D [13]. The minimum cardinality of such a set is the *fair domination number* $\gamma_F(G)$. This invariant captures equitable access: every vertex is dominated by the same number of vertices. As mentioned in [39] fair domination promotes a more balanced influence and provides a useful tool for resource allocation.

Meeting all clients' needs is a desirable, but often unrealistic, requirement, mostly because of economical restrictions. In such cases, requiring that just a fraction of the demand must be serviced is more beneficial.

Therefore, the focus falls on the dominated vertices, that might not be the whole set of vertices. This motivates the use of the concept of *partial domination* in graphs. For $\alpha \in [0, 1]$, a set $S \subseteq V$ is a *α-partial dominating set* of a graph G if the ratio $|B|/|V|$ is at least α, where B is the set of vertices of S together with all the vertices adjacent to a vertex of S. The *α-partial domination number* $\gamma_\alpha(G)$ equals the minimum cardinality of a α-partial dominating set. This concept was independently introduced in [14, 18], and mentioned to have the capability for being used in micromobility models in [43].

Exact solutions for the partial domination problem are not always available. In [43], authors presented two genetic algorithm heuristics and one artificial bee colony heuristic to solve the minimum general partial dominating set problem. Although their algorithms were not applied directly to micromobility datasets, the authors highlighted their suitability for facility location in urban networks.

1.1.5 Fuzzy graphs

Zadeh [54] introduced the concept of fuzzy sets, laying the foundation for handling vagueness in mathematical structures. When vertices, edges, and their relationships in a graph exhibit ambiguous or imprecise information, representing them through a fuzzy graph becomes essential. The early developments in this field were led by Rosenfeld [44], whose work was later extended by researchers such as Mordeson [37] and Bhattacharya [9].

The concept of domination in the context of fuzzy graphs was later introduced by Somasundaram [51]. In the 21st century, Akram [2] extended this line of research by developing the notion of bipolar fuzzy graphs and investigating some of their properties. The same authors further studied irregular bipolar fuzzy graphs of various types and subsequently introduced the concept of regular bipolar fuzzy graphs [3]. Applications can be found in diverse domains, including decision-making, and social networks [34, 41, 50].

Alqahtani [5] applies domination in neutrosophic fuzzy directed graphs to decide where to install charging stations for electrical vehicles. Under this model, the candidate sites are modeled as vertices with neutrosophic values that represent suitability, where edges encode accessibility and service influence. By computing dominating sets, the method gives an approximation to a solution even when evaluations are uncertain or contradictory. The same idea could be used for bike or scooter stations by redefining the vertex attributes to reflect demand density, connectivity and land-use compatibility.

Nithyanandham et al. [40] applied the concept of domination in bipolar intuitionistic fuzzy graphs to identify flood-prone regions. In its model, both positive and negative evidence are incorporated, showing how domination can highlight sites that require intervention despite contradictory evidences. This dual perspective is useful in mobility planning, where each candidate site may combine advantages (like high demand or centrality) with certain disadvantages (like hard traffic or limited space). Through domination, it becomes possible to identify robust sets of sites that balance competing advantages and disadvantages.

Akram and Waseem [4] study domination in bipolar fuzzy graphs from a broader perspective and demonstrated its applicability to facility location problems. Their examples illustrate how domination can be used to select a small set of facilities that adequately represent all alternatives under uncertainty. For transportation systems, this idea naturally translates into selecting docking sites that cover demand while minimizing cost.

Gong [25] explores several domination numbers in bipolar fuzzy graphs, providing numerical examples for planning tasks in an urban environment. While it is not exactly devoted to transportation, the algorithms could be directly redefined to compute the locations of stations once demand and accessibility values are codified as a fuzzy value.

Finally, Sudakov and Zhukov [52] introduce fuzzy domination graphs in the context of decision support. Although their case studies are generic, the framework can be easily adapted to order candidate sites for bike or scooter stations, where dominating subsets offer a natural solution to covering demand under limited resources.

In summary, the contributions discussed demonstrate that domination in fuzzy graphs offers a suitable approach for decision-making under uncertainty. Existing models for charging stations, disaster management, and general facility placement can be adapted directly to optimize micromobility infrastructure. By representing candidate sites as fuzzy vertices and service overlaps as edges, the framework of domination theory provides efficient and interpretable rules for selecting station locations in complex urban environments.

1.2 Complexity of the problem

Most domination parameters are NP-hard to compute, even on restricted graph classes [29]. Nevertheless, approximation algorithms and heuristics have been widely studied. Greedy algorithms for the set cover problem yield constant-factor approximations for dominating set. For distance and k-domination, metaheuristic approaches such as tabu search, simulated annealing, and genetic algorithms have been applied [17, 33]. For partial domination metaheuristic approaches were investigated in [43] and parallel algorithms in [30]. Corcoran et al. [17] studied heuristic algorithms for k-domination on street networks. These approaches are essential in large-scale urban networks where exact optimization is intractable.

1.3 Conclusions

Graph-theoretic approaches, particularly domination and its variants, provide effective tools for optimizing micromobility facility locations, balancing coverage, accessibility, and equity. Fuzzy graph models enhance this framework by handling uncertainty and conflicting evidence, enabling robust decision-making.

Despite the extensive theoretical development, several research gaps remain in applying domination invariants to micromobility. In particular, the integration of domination-based models with capacity constraints and stochastic demand has received little attention. Moreover, dynamic and temporal aspects, such as time-dependent isolation numbers or rolling-horizon fair domination, have yet to be explored. Addressing these gaps offers a promising research direction at the intersection of graph theory and micromobility systems.

Bibliography I

[1] Taras Agryzkov, Leandro Tortosa, José F Vicent, and Richard Wilson. "A centrality measure for urban networks based on the eigenvector centrality concept". In: *Environment and Planning B: Urban Analytics and City Science* 46.4 (2019), pp. 668–689. doi: 10.1177/2399808317724444.

[2] Muhammad Akram. "Bipolar fuzzy graphs". In: *Information Sciences* 181.24 (2011), pp. 5548–5564. doi: 10.1016/j.ins.2011.07.037.

[3] Muhammad Akram and Wiesław A. Dudek. "Regular bipolar fuzzy graphs". In: *Neural Computing and Applications* 21.Suppl 1 (2012), pp. 197–205. doi: 10.1007/s00521-011-0772-6.

[4] Muhammad Akram and Neha Waseem. "Novel applications of bipolar fuzzy graphs to decision making problems". In: *Journal of Applied Mathematics and Computing* 56.1 (2018), pp. 73–91. doi: 10.1007/s12190-016-1062-3.

[5] Mohammed Aeyed M. Alqahtani. "Determining electrical vehicle charging stations using dominance in neutrosophic fuzzy directed graphs". In: *European Journal of Pure and Applied Mathematics* 18.1 (2025), p. 5675. doi: 10.29020/nybg.ejpam.v18i1.5675.

[6] Javad J.C. Aman, Myriam Zakhem, and Janille Smith-Colin. "Towards equity in micromobility: Spatial analysis of access to bikes and scooters amongst disadvantaged populations". In: *Sustainability* 13.21 (2021), p. 11856. doi: 10.3390/su132111856.

[7] Georgia Ayfantopoulou, Josep Maria Salanova Grau, Zisis Maleas, and Alexandros Siomos. "Micro-mobility user pattern analysis and station location in Thessaloniki". In: *Sustainability* 14.11 (2022). doi: 10.3390/su14116715.

[8] Simona De Bartolomeo, Michele Ottomanelli, and Leonardo Caggiani. "An equity parking area location model for transition from dockless to docked shared micromobility systems". In: *npj Sustainable Mobility and Transport* 2 (2025), p. 23. doi: 10.1038/s44333-025-00038-4.

[9] Prabir Bhattacharya. "Some remarks on fuzzy graphs". In: *Pattern Recognition Letters* 6.5 (1987), pp. 297–302. doi: 10.1016/0167-8655(87)90012-2.

[10] Phillip Bonacich. "Factoring and weighting approaches to status scores and clique identification". In: *Journal of Mathematical Sociology* 2.1 (1972), pp. 113–120. doi: 10.1080/0022250X.1972.9989806.

[11] Tulsi Bora and Arun Kumar Baruah. "Basic centrality measures on urban transport network nodes". In: *Journal of Mathematical and Computational Science* 11.5 (2021), pp. 5433–5446. doi: 10.28919/jmcs/6034.

[12] Leonardo Caggiani, Aleksandra Colovic, and Michele Ottomanelli. "An equality-based model for bike-sharing stations location in bicycle-public transport multimodal mobility". In: *Transportation Research Part A: Policy and Practice* 140 (2020), pp. 251–265. doi: 10.1016/j.tra.2020.08.015.

[13] Yair Caro, Adriana Hansberg, and Michael Henning. "Fair domination in graphs". In: *Discrete Mathematics* 312.19 (2012), pp. 2905–2914. doi: 10.1016/j.disc.2012.05.006.

[14] Benjamin M. Case, Stephen T. Hedetniemi, Renu C. Laskar, and Drew J. Lipman. "Partial domination in graphs". In: *Congressus Numerantium* 228 (2017), pp. 85–95.

[15] Richard Church and Charles ReVelle. “The maximal covering location problem”. In: *Papers of the Regional Science Association* 32 (1974), pp. 101–118. doi: 10.1007/BF01942293.

[16] Aleksandra Colovic, Luigi Pio Prencipe, Nadia Giuffrida, and Michele Ottomanelli. “A multi-objective model to design shared e-kick scooters parking spaces in large urban areas”. In: *Journal of Transport Geography* 116 (2024), paper no. 103823. doi: 10.1016/j.jtrangeo.2024.103823.

[17] Padraig Corcoran and Andrei Gagarin. “Heuristics for k-domination models of facility location problems in street networks”. In: *Computers & Operations Research* 133 (2021), p. 105368. doi: 10.1016/j.cor.2021.105368.

[18] Angsuman Das. “Partial domination in graphs”. In: *Iranian Journal of Science and Technology, Transactions A: Science* 43 (2019), pp. 1713–1718. doi: 10.1007/s40995-018-0618-5.

[19] Paul DeMaio. “Bike-sharing: History, impacts, models of provision, and future”. In: *Journal of Public Transportation* 12.4 (2009), pp. 41–56. doi: 10.5038/2375-0901.12.4.3.

[20] Lukas Dijkstra, Andrei Gagarin, Padraig Corcoran, and Rhyd Lewis. “Digraphs and k-domination models for facility location problems in road networks: greedy heuristics”. In: *Proceedings of the 11th International Network Optimization Conference* (2024). doi: 10.48786/inoc.2024.05.

[21] Elliot Fishman, Simon Washington, and Narelle Haworth. “Bike share: a synthesis of the literature”. In: *Transport Reviews* 33.2 (2013), pp. 148–165. doi: 10.1080/01441647.2013.775612.

[22] Linton C. Freeman. “A set of measures of centrality based on betweenness”. In: *Sociometry* (1977), pp. 35–41. doi: 10.2307/3033543.

[23] Linton C. Freeman. “Centrality in social networks conceptual clarification”. In: *Social Networks* 1.3 (1978), pp. 215–239. doi: 10.1016/0378-8733(78)90021-7.

[24] Andrei Gagarin and Padraig Corcoran. “Multiple domination models for placement of electric vehicle charging stations in road networks”. In: *Computers & Operations Research* 96 (2018), pp. 69–79. doi: 10.1016/j.cor.2018.03.014.

[25] Shu Gong, Gang Hua, and Wei Gao. “Domination of bipolar fuzzy graphs in various settings”. In: *International Journal of Computational Intelligence Systems* 14.162 (2021), pp. 1–14. doi: 10.1007/s44196-021-00011-2.

[26] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning. *Topics in Domination in Graphs*. Vol. 64. Springer, 2020.

[27] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning. *Structures of Domination in Graphs*. Vol. 66. Springer, 2021.

[28] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning. *Domination in Graphs: Core Concepts*. Springer, 2023.

[29] Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater. *Fundamentals of Domination in Graphs*. CRC Press, 2013.

[30] Weizhi Hong, Yingli Ran, and Zhao Zhang. “Parallel algorithms for minimum general partial dominating set and maximum budgeted dominating set in unit disk graph”. In: *Theoretical Computer Science* 932 (2022), pp. 13–20. doi: 10.1016/j.tcs.2022.07.039.

[31] Yujie Hu, Yongping Zhang, David Lamb, Mingming Zhang, and Peng Jia. “Examining and optimizing the BCycle bike-sharing system – A pilot study in Colorado, US”. In: *Applied Energy* 247 (2019), pp. 1–12. doi: 10.1016/j.apenergy.2019.04.007.

[32] Dipti S. Joshi, Sridhar Radhakrishnan, and Chandrasekheran Narayanan. "A fast algorithm for generalized network location problems". In: *Proceedings of the 1993 ACM/SIGAPP Symposium on Applied Computing: States of the Art and Practice*. 1993, pp. 701–708. doi: 10.1145/162754.167181.

[33] Ralf Klasing and Christian Laforest. "Hardness results and approximation algorithms of k-tuple domination in graphs". In: *Information Processing Letters* 89.2 (2004), pp. 75–83. doi: 10.1016/j.ipl.2003.10.004.

[34] Ali N. A. Koam, Muhammad Akram, and Peide Liu. "Decision-making analysis based on fuzzy graph structures". In: *Mathematical Problems in Engineering* 2020.1 (2020), paper no. 6846257. doi: 10.1155/2020/6846257.

[35] Richard Mix, Ricardo Hurtubia, and Sebastián Raveau. "Optimal location of bike-sharing stations: A built environment and accessibility approach". In: *Transportation Research Part A: Policy and Practice* 160 (2022), pp. 126–142. doi: 10.1016/j.tra.2022.03.022.

[36] Stephen J. Mooney, Kate Hosford, Bill Howe, An Yan, Meghan Winters, Alon Bassok, and Jana A. Hirsch. "Freedom from the station: Spatial equity in access to dockless bike share". In: *Journal of Transport Geography* 74 (2019), pp. 91–96. doi: 10.1016/j.jtrangeo.2018.11.009.

[37] John N. Mordeson and Peng Chang-Shyh. "Operations on fuzzy graphs". In: *Information Sciences* 79.3 (1994), pp. 159–170. doi: 10.1016/0020-0255(94)90116-3.

[38] Herlina Napitupulu, Ema Carnia, Nursanti Anggriani, and Asep K. Supriatna. "Centrality measures in transportation networks for Unpad campus route". In: *Journal of Physics: Conference Series*. Vol. 1722. 1. IOP Publishing. 2021, paper no. 012063. doi: 10.1088/1742-6596/1722/1/012063.

[39] G. Navamani and M.A. Reena Mercy. "Optimizing subdivisions on fair domination in Petersen graph structures". In: *Advances in Nonlinear Variational Inequalities* 28.3 (2025). doi: 10.52783/anvi.v28.2845.

[40] Deva Nithyanandham, Felix Augustin, Samayan Narayanamoorthy, Ali Ahmadian, Dumitru Balaenu, and Daekook Kang. "Bipolar intuitionistic fuzzy graph based decision-making model to identify flood vulnerable region". In: *Environmental Science and Pollution Research* 30.60 (2023), pp. 125254–125274. doi: 10.1007/s11356-023-27548-3.

[41] Madhumangal Pal, Sovan Samanta, and Ganesh Ghorai. "Few applications of fuzzy graphs". In: *Modern Trends in Fuzzy Graph Theory*. Singapore: Springer Singapore, 2020, pp. 275–305. doi: 10.1007/978-981-15-8803-7_10.

[42] Mangal Pati, Uma Shankar, Prodip Karmakar, and Shambhu Charan Barman. "Optimizing urban resource allocation using minimum dominating sets in interval graphs". In: *International Journal for Multidisciplinary Research* 7.3 (2025). doi: 10.36948/ijfmr.2025.v07i03.44892.

[43] Mohd Danish Rasheed and Alok Singh. "Solution of minimum general partial dominating set problem through heuristic approaches". In: *Computers and Electrical Engineering* 125 (2025), p. 110427. doi: 10.1016/j.compeleceng.2025.110427.

[44] Azriel Rosenfeld. "Fuzzy graphs". In: *Fuzzy Sets and their Applications to Cognitive and Decision Processes*. Ed. by Lotfi A. Zadeh, King-Sun Fu, Kokichi Tanaka, and Masamichi Shimura. Academic Press, 1975, pp. 77–95. doi: 10.1016/B978-0-12-775260-0.50008-6.

[45] B. A. Sabarish, B. Kailassh, Kirthana Baktha, and Y. Janaki. "Recommendations of location for facilities using domination set theory". In: *2017 International Conference on Communication and Signal Processing*. 2017, pp. 1540–1544. doi: 10.1109/ICCSP.2017.8286646.

[46] Gert Sabidussi. "The centrality index of a graph". In: *Psychometrika* 31.4 (1966), pp. 581–603. doi: 10.1007/BF02289527.

[47] Ricardo Sandoval, Caleb Van Geffen, Michael Wilbur, Brandon Hall, Abhishek Dubey, William Barbour, and Daniel B. Work. "Data-driven methods for effective micromobility parking allocation". In: *Transportation Research Interdisciplinary Perspectives* 10.1–2 (2021), paper no. 100368. doi: 10.1016/j.trip.2021.100368.

[48] David Schoch. *Periodic Table of Network Centrality*. 2016. url: http://schochastics.net/sna/periodic.html (visited on 09/12/2025).

[49] Susan A. Shaheen, Stacey Guzman, and Hua Zhang. "Bikesharing in Europe, the Americas, and Asia: Past, Present, and Future". In: *Transportation Research Record* 2143.1 (2010), pp. 159–167. doi: 10.3141/2143-20.

[50] Muzzamal Sitara, Muhammad Akram, and Muhammad Yousaf Bhatti. "Fuzzy graph structures with application". In: *Mathematics* 7.1 (2019). doi: 10.3390/math7010063.

[51] A. Somasundaram and S. Somasundaram. "Domination in fuzzy graphs – I". In: *Pattern Recognition Letters* 19.9 (1998), pp. 787–791. doi: 10.1016/S0167-8655(98)00064-6.

[52] Vladimir Sudakov and Alexander Zhukov. "Fuzzy domination graphs in decision support tasks". In: *Mathematics* 11.13 (2023), paper no. 2837. doi: 10.3390/math11132837.

[53] Jianwei Yue, Yingqiu Long, Shaohua Wang, and Haojian Liang. "Optimization of shared electric scooter deployment stations based on distance tolerance". In: *ISPRS International Journal of Geo-Information* 13.5 (2024), paper no. 147. doi: 10.3390/ijgi13050147.

[54] Lotfi A. Zadeh. "Fuzzy sets". In: *Information and Control* 8.3 (1965), pp. 338–353. doi: 10.1016/S0019-9958(65)90241-X.

Chapter II

Bike-Sharing Systems

Bike-sharing systems (BSS) have become an essential component of sustainable urban mobility by reducing congestion, complementing public transport, and promoting eco-friendly travel behaviour. Their rapid development, from early docked models to modern free-floating systems with e-bikes, has introduced a range of design and operational challenges. Among these, the rebalancing problem — redistributing bicycles across stations to counter supply-demand imbalances — remains one of the most critical at the operational level. In this chapter, we provide a survey of the static bike rebalancing problem (SBRP), which focuses on relocation during off-peak hours using a dedicated fleet of vehicles. We discuss problem variants such as multiple depots, heterogeneous fleets or the integration of e-bikes and review mathematical models, exact and heuristic solution methods. Finally, we highlight that addressing the large-scale real-world challenges faced by bike-sharing systems requires more comprehensive approaches, often combining different methods to achieve good-enough solutions.

Keywords: Static rebalancing, bike-sharing systems, SBRP, vehicle routing, algorithms, mixed integer linear programming, complexity.

II

Static Rebalancing Problem in Urban Bike-Sharing Systems

Carmen-Ana Domínguez-Bravo, University of Cádiz, Spain
Hanna Furmańczyk, University of Gdańsk, Poland
Manuel Munoz-Marquez, University of Cádiz, Spain
María José Pereira Sáez, University of A Coruña, Spain
María González Taboada, University of A Coruña, Spain

Urban bike-sharing systems have emerged as key components of sustainable urban mobility strategies. By providing flexible access to bicycles without ownership, they aim to reduce traffic congestion and promote environmentally friendly travel behaviour complementing the traditional transport. A bike sharing system (BSS) is a transportation service that enables users to rent bikes for short-term use, typically within urban environments. It offers a sustainable and flexible mobility option, allowing users to pick up and return bikes at parking stations.

The *first-generation* of bike sharing systems started in 1965 in the city of Amsterdam with the "White Bikes". As detailed in [12], in this case ordinary bikes painted in white were put at disposal for public use. However, this program collapsed within days. Afterwards, the *second-generation* of bike-sharing programs started in 1991 in Denmark, where they incorporated stations and a coin-based system to pay for the use of the bikes. In this case, many improvements were made but the anonymity of users continued to encourage theft. The *third-generation* of bike-sharing programs started in 1996 in England, where they incorporated a user tracking system. The interest of this mean of transport exploded in 2007, when Velib' was launched in Paris. As mentioned in [2], Velib' system was largely bigger than previous bike sharing systems. Since then, these systems have been implemented in a large number of cities worldwide. We refer the interested reader to [12] to have a complete history of these three bike-sharing systems. The third generation was also known as station-based bike sharing (SBBS) systems. One of the new considerations in the *fourth-generation* systems was to eliminate the need for docks or fixed stations. Therefore, the fourth generation of bike-sharing systems are also known as free-floating bike sharing (FFBS) systems as mentioned in [35, 43]. Dated from 2014 in China, these new systems also included features like GPS tracking, electric bikes (e-bikes), and digital payment options to enhance user experience and operational efficiency.

A bike-sharing system can be described through three main components. The first is the **bikes**, which may be purely mechanical, electric (e-bikes), or a combination of both, depending on the system. Moreover, as [31] reflects, in some specific BSS, multiple types of bikes are considered (e.g., bikes with one, two, or three seats and those with a child-seat). Second, the **stations**, which can either be fixed with limited capacity (docked systems) or flexible, allowing bikes to be placed within designated areas without specific capacity constraints (dock-less systems). Third, the system includes one or several **depots**, which are used for performing maintenance work and/or storing extra bicycles.

In some cases, both repositioning and repairing of bikes are done at the same time, whereas in other systems, special collection routes for out-of-service bikes are planned. Furthermore, when electrical bikes are available, some BSS have also specific charging locations. In this chapter, we focus our attention on systems where the rebalancing process is separated from any other maintenance activities.

There are different sharing-schemes in shared mobility systems. We will focus on the case of one-way systems, which allow users to pick up a bike at any station and return it to any other station, facilitating flexible journeys. On the contrary, two-way systems require users to return the element to the same station where it was picked up, limiting flexibility but potentially simplifying logistics.

As expected, designing an efficient bike-as-a-service network for citizens is a complex task, with municipalities and bike-sharing companies facing numerous challenges across multiple levels. The article in [42] review and systematically classify the existing literature of bicycle-sharing problems at strategic, tactical, and operational decision levels. Different problems arise at each level, some of them are listed below.

- *From the strategic level*: the network design (the location of the depot and bikes stations) and the sizing problem (the optimal fleet dimensioning in terms of vehicles and bikes).
- *From the tactical level*: the station inventory problem (distribution and capacities), the forecasting of the demand, the study of pricing policies (incentive programs to influence users behaviour), and the coordination with other transport modes (multimodal travelling).
- *From the operational level*: the rebalancing problem (relocate bikes among the stations) and the battery swapping problem.

We refer the interested readers to some complete reviews about bike sharing systems: [12, 16, 41, 42, 47]. The mathematical problems arising from these systems have received increasing attention. Some interesting reviews on shared mobility systems from the mathematical modeling point of view can be found in [27, 28].

In the next section, we focus on the rebalancing problem, which arises from imbalances between supply and demand caused by user behavior in bike-sharing systems. The objective is to develop efficient methods for redistributing bikes among stations using a fleet of vehicles, thereby preventing both shortages and surpluses.

2.1 The Static Bike Rebalancing Problem

Despite the success and rapid expansion of the BSS in the last century in many cities all over the world, these systems face an intrinsic operational challenge: the uneven distribution of bicycles across time and space. As user demand is highly unbalanced and varies between locations and periods of the day, some stations or service zones often experience shortages of available bikes, while others face excess. In docked systems, the problem is compounded by the limited number of docking points, which may lead to stations being full or empty, thus reducing service availability and user satisfaction.

To mitigate these imbalances, operators must implement relocation strategies that restore a more desirable system state. Broadly, two categories of approaches can be distinguished: operator-based rebalancing, where dedicated service vehicles redistribute bicycles, and user-based incentives, where customers are encouraged to pick up or return bikes at specific locations. Operator-based relocation is further divided into static and dynamic rebalancing. Static rebalancing is typically performed during off-peak hours (often at night), when the system is inactive or user activity is minimal, and the goal is to prepare the system for the next operating period. In contrast, dynamic rebalancing takes place while the system is active and requires continuous adjustments in response to real-time demand fluctuations.

This chapter focuses primarily on the Static Bike Rebalancing Problem (SBRP), which can be described as the task of designing vehicle routes and loading/unloading operations to achieve a target distribution of bicycles at stations, while minimizing operational costs. That is, given a single commodity initially distributed among the vertices of a graph and a fleet of capacitated vehicles, the objective is to determine optimal routes that redistribute the commodity to achieve the target distribution, balancing all the vertices.

SBRP represents a challenging combinatorial optimization problem that is related to classical problems in transportation and logistics, such as vehicle routing and network flows, but also exhibits specific features. These include heterogeneous demand across stations, the interdependence of pick-up and delivery operations, capacity limitations of relocation vehicles, depot locations, and time restrictions on operations.

Next, we briefly review the literature related to the static rebalancing problem, Mixed Integer Linear Programming (MILP) formulations and solution methods.

2.1.1 Variants of the Bike Rebalancing Problem

Static rebalancing takes place at night, when traffic is low and user activity is assumed to be negligible. As mentioned earlier, users cannot interact with the bikes during pickup and delivery operations. The initial state of the system is assumed to be known and fixed, and the objective of the rebalancing is to move the system toward a predefined target state. In contrast, in the dynamic repositioning strategies, the bike sharing system is being used at the same time that bikes are relocated. Usually, these strategies are performed several times during the day. In [46], the authors propose a hybrid rebalancing framework that simultaneously integrates dynamic and static rebalancing.

Owing to the practical nature of the bicycle rebalancing problem, numerous variants have emerged in the literature, reflecting a range of different considerations. Our objective in this section is to provide a brief overview of these variants. For ease of reading, we will include summary tables throughout this section (see Tables 2.1 and 2.2) where each row corresponds to an article, mentioning the year of publication and the first author. We use a short dash (-) to indicate that the column aspect is not mentioned in the corresponding paper. When some specific option is not allowed for the modeled case, we represent it with a cross (×).

Table 2.1 presents a broad classification of the models proposed in several studies on the Static Rebalancing Problem. We categorize the papers based on how each model estimates demand, the number of bike types considered, and whether docking stations are required. By 'demand,' we refer to the number of bikes required at each station at the conclusion of the rebalancing process. When the demand at each station is represented by random variables with associated probability distributions, it is referred to as stochastic demand [11]. In contrast, if the demand is known in advance or estimated from historical data, it is termed deterministic demand. The distinction between point and interval refers to how the objective is achieved: whether an acceptable range of values is considered or a specific number is targeted for the distribution of bikes between stations.

Today, most bike-sharing systems (BSS) include both electric and mechanical bikes. When the bikes are similar, or only a single type is present, we classify the system as a single-bike model. If different bike types requiring distinct transport vehicle capacities are considered, as in [31], we classify it as a multiple-bike model. Even in studies that distinguish between damaged and usable bikes, such as [1] and [48], we did not find any work specifying a required number or percentage for each bike type when similar electric and mechanical bikes are available.

Bike-sharing systems are generally classified into two main types: docked and dockless. In a station-based system (SBSS), users can pick up and return bikes only at designated stations, whereas in a dockless system (FFBS), bikes can be picked up and dropped off anywhere.

The information presented in Table 2.2 summarizes the different options for depot distribution, the number and types of transport vehicles, and the requirements for rebalancing routes. The options single (S) and multiple (M) refer to the cases in which the model has only one depot or several ones. The “Empty" option reflects the cases in which transport vehicles begin the balancing route at the main depot without bikes, so the rebalancing is made only by redistributing the already existing bikes at stations. If only a single vehicle is used, it is denoted as single (S). When a fleet of vehicles is employed, it is classified as multiple (M), with the vehicles being either identical or different (Diff). Regarding route design, vehicles may be required to start and end at the depot (Cycle), while in other cases this is not mandatory. In some models, visiting all stations is compulsory (All), whereas in others, a station need not be visited if it already has the desired number of bikes. Additionally, multiple visits to the same station may be permitted (M), while in other cases only a single visit per station is allowed (S).

At the station level, vehicles may occasionally visit the same station multiple times and even use stations as temporary bike storage. As noted in [2], the rebalancing problem can be classified into two variants: preemptive and non-preemptive. In the preemptive variant (Preemptive), bicycles can be temporarily offloaded at intermediate locations along the route before being delivered to their final destinations.

In some approaches, the repositioning and maintenance optimization problems are addressed simultaneously [24], whereas in other systems special collection routes for out-of-service bikes are planned. Moreover, bicycles stored at the depot, such as those that were damaged and subsequently repaired, can also be utilized for rebalancing.

Table 2.2: Static Rebalancing problem classification II.

		Depot			Vehicle			Routing							
Year	Authors	S	M	Empty	S	M	Diff	Cycle		All		M	S	Preemptive	
2011	Benchimol et al. [2]	-	-		✓			✓	×		×	✓		✓	
2013	Chemla, Meunier, and Calvo [4]	✓			✓			✓			×	✓		✓	
	Rainer-Harbach et al. [37]		✓	✓		✓	✓		×		×	✓			×
	Raviv, Tzur, and Forma [39]	✓				✓	✓	✓		✓		✓		✓	
2014	Chira et al. [7]		✓			✓		✓		✓			✓		×
	Dell'Amico et al. [9]	✓				✓		✓		✓			✓		×
	Erdoğan, Laporte, and Calvo [15]	✓			✓			✓			×		✓		×
	Ho and Szeto [22]	✓			✓				×		×		✓		×
2015	Erdoğan, Battarra, and Calvo [14]	✓			✓			✓			×	✓		✓	
	Forma, Raviv, and Tzur [17]	✓				✓		✓			×		✓		×
	Rainer-Harbach et al. [38]	✓		✓		✓		✓			×	✓		✓	
2016	Alvarez-Valdes et al. [1]		✓			✓			×		×	✓			×
	Dell'Amico et al. [10]	✓				✓		✓		✓			✓		×
	Gaspero, Rendl, and Urli [20]		✓			✓		✓			×	✓			×
	Kadri, Kacem, and Labadi [25]	✓			✓				×	✓			✓		×
	Li et al. [31]	✓			✓			✓			×		✓		×
	Szeto, Liu, and Ho [44]	✓			✓			✓			×		✓		×
2017	Cruz et al. [8]	✓	✓	✓	✓			✓			×	✓			×
	Ho and Szeto [23]	✓				✓		✓			×	✓		✓	
	Pal and Zhang [35]	✓			✓	✓			×		×	✓			×
	Schuijbroek, Hampshire, and Van Hoeve [40]		✓			✓			×		×	✓		✓	
2018	Dell'Amico et al. [11]	✓				✓		✓		✓			✓		×
	Leclaire and Couffin [29]			✓		✓			×			✓			×
	Liu, Szeto, and Ho [32]		✓	✓		✓	✓		×		×	✓		✓	
	Szeto and Shui [45]	✓		✓		✓		✓		-	-		✓		×
2019	Lahoorpoor et al. [26]	✓			✓			✓			×	✓			×
	Wang and Wu [48]	✓		✓		✓		✓		✓		✓			×
2020	Du et al. [13]		✓	✓		✓	✓		×	✓		✓		✓	
2021	Li and Liu [30]	✓		✓		✓		✓			×		✓		×
	Ma et al. [34]	-	-			✓	✓	-	-	✓		✓		-	-
2023	Lu, Gao, and Huang [33]	✓				✓		✓		✓		✓		-	-
2024	Yu et al. [50]		✓			✓		✓			×	✓		-	-

Traditionally, the rebalancing strategies have been classified as operator-based or user-based strategies. In the first case, the system operator relocates bikes using their fleet of vehicles. In the second case, the companies apply user incentives in order to modify users behaviors, promoting the use of certain strategic stations helping to rebalance the system. Some studies consider rebalancing incentives (i.e., static and dynamic pricing schemes), for example, when riders are offered discount to pick up/drop off bikes from/at nearby stations that are full/empty or expected to become full/empty in the near future ([18, 21, 36]). It is not clear whether both strategies are equally effective, as highlighted by [19] in their study of the bike-sharing system in the Sacramento region (California, USA).

In the literature, different objective functions have been considered: travel cost, total redistribution cost (travel and loading/unloading), total absolute deviation from the target value, total unmet demand, makespan of the rebalancing fleet, etc.

In [35], two categories of rebalancing are distinguished: partial and complete. Partial rebalancing allows some nodes to fall short of the target inventory, whereas complete rebalancing requires that all nodes meet their target levels. Since complete rebalancing may be infeasible in certain instances, allowing partial rebalancing can be useful for finding feasible solutions.

Numerous emerging variants of the SBRP consider factors such as demand uncertainty [3], demand estimation under uncertainty [1, 11], or the simultaneous optimization of rebalancing and battery-swapping strategies [49].

2.1.2 Complexity of the Bike Relocation Problem

The SBRP entails a Vehicle Routing Problem (VRP) that additionally requires determining the quantities of bikes to be picked up and dropped off. As explained, for instance, in [9], the SBRP can be regarded as a special case of the One-Commodity Pickup-and-Delivery Capacitated VRP, where vehicles transport bikes from certain nodes to others within the network. A key distinction is that the number of bikes to be transported between stations is a decision variable. The problem can also be interpreted as a variant of Many-to-Many VRPs, in which a request may involve multiple origins (pickup stations) and multiple destinations (drop-off stations).

As noted in [3, 5, 9], the SBRP is NP-hard, even in the single-vehicle case. When the bike repositioning problem involves only one vehicle, it can be viewed as a generalization of the One-Commodity Pickup-and-Delivery Traveling Salesman Problem [15]. Therefore, MILP formulations are intractable to solve large instances. Hence, as detailed in the following section, efficient algorithms are usually developed in the literature to solve real-world case scenarios.

2.1.3 Formulations and solution methods

The main objective of this section is to review the existing literature on the static rebalancing problem, with an emphasis on mixed integer programming formulations and solution procedures devised specifically for this problem. As in the previous sections, we will use summary tables along the section with the same notation as before (see Tables 2.3, 2.4 and 2.5).

Developing a suitable MILP formulation for a real case application is not an easy task. For instance, in [29] the authors directly propose a method to construct an MILP formulation for the rebalancing problem based on the use of Unified Modeling Language graphical class diagram with the problem properties. Most of the MILP formulations for the SBR problem are summarized in Table 2.3 according to the number of indices used (2, 3, 4 or 5-index formulation). In [9], one of the two-index formulation proposed is based on the Multiple Traveling Salesman Problem (m-TSP) formulation. Some of the three-index formulations are flow-based formulation with Miller-Tucker-Zemlin (MTZ) subtour elimination constraints. Due to the complexity of the problem when solving real world instances, the two-index formulations are the most commonly used in the literature.

Various other methodologies have also been explored in the literature for this problem, including constraint programming [20], nonlinear programming [30, 38], stochastic models [6, 11, 34, 50], and cluster-based approaches [26].

Table 2.3: MILP formulations for the Static Rebalancing problem.

		# Index Formulation				
Year	**Authors**	2	3	4	5	**Comments**
2013	Chemla, Meunier, and Calvo [4]			✓		
	Raviv, Tzur, and Forma [39]		✓	✓		
2014	Chira et al. [7]		✓			
	Dell'Amico et al. [9]	✓				Based on (m-TSP) and Flow model
	Erdoğan, Laporte, and Calvo [15]	✓				
	Ho and Szeto [22]	✓				
2015	Erdoğan, Battarra, and Calvo [14]		✓			
2016	Dell'Amico et al. [10]	✓				Based on Dellamico2014
	Kadri, Kacem, and Labadi [25]	✓				
	Li et al. [31]			✓		
	Szeto, Liu, and Ho [44]	✓				
2017	Ho and Szeto [23]		✓			Revised from Raviv2013
	Pal and Zhang [35]	✓				
	Schuijbroek, Hampshire, and Van Hoeve [40]			✓		Based on Raviv2013
2018	Liu, Szeto, and Ho [32]			✓		
	Szeto and Shui [45]		✓			
2019	Wang and Wu [48]	✓				
2020	Du et al. [13]				✓	
2023	Lu, Gao, and Huang [33]	✓				

Exact solution methods are enumerated in Table 2.4. We give a brief description of the methodology used as well as the data used on the paper. Note that the work of [20] focuses on constraint programming, while the work of [11] uses stochastic programming.

Table 2.4: Exact solution approaches.

Year	**Authors**	**Description**	**Instances**
2013	Raviv, Tzur, and Forma [39]	Arc deletion to reduce variables	Real-world data
2014	Dell'Amico et al. [9]	Branch-and-cut algorithm.	Real-world data
	Erdoğan, Laporte, and Calvo [15]	Branch-and-cut algorithm with Bender's cuts	Literature instances
2015	Erdoğan, Battarra, and Calvo [14]	Separation method based on Bender's cuts	Literature instances
2016	Gaspero, Rendl, and Urli [20]	Branch-and-bound algorithm.	Real-world data
2018	Dell'Amico et al. [11]	Multi-cut L-Shaped method and Branch-and-cut	Adapt Real-world data

Most of the exact solution approaches are limited to small size instances, while large instances require the use of non-exact methods. The non-exact methods proposed to this problem are summarized in Table 2.5, including approximation methods (A), heuristics (H), meta-heuristic (M) and math-heuristic (Ma). We give a brief description of the proposed algorithm in the last column of Table 2.4. As observed, a wide variety of algorithms have been employed, ranging from branch-and-cut methods to genetic algorithms and other nature-inspired optimization techniques. In some approaches, the original problem is simplified by decomposing it into more manageable sub-problems, which are solved in separate steps. Some of these methods yield sufficiently accurate solutions for larger real-world instances within a reasonable amount of time. We particularly highlight the works of [17] and [40], in which the original network is first clustered into distinct zones, and routing decisions are subsequently made independently within each zone.

Today, bike-sharing systems are implemented in most major cities worldwide. Several real-world instances have been used in the literature to evaluate the performance of proposed methods, including Barcelona [7], Paris [17, 22, 39], Washington [6, 39, 40], and Shanghai [33, 50], among others. A comprehensive collection of real-world instances with real data from different cities is presented in [9, 10].

2.2 Conclusions

The static rebalancing problem is a central operational challenge in bike-sharing systems, directly affecting service quality and system sustainability. Over the past two decades, research has produced a variety of mathematical models and algorithmic approaches, yet significant gaps remain in addressing large-scale, uncertain, and heterogeneous environments. Bridging these gaps will require hybrid methods that combine optimization, heuristics, and data-driven prediction, ensuring that theoretical advances translate into practical improvements for real-world systems.

The comparative perspective provided in our tables further illustrates the diversity and limitations of existing approaches. Table 2.1 highlights how modeling choices vary with respect to demand estimation, bicycle heterogeneity, and the presence or absence of docking stations. Table 2.2 reflects operational factors such as depot configurations, fleet composition, and routing requirements. Tables 2.3-2.5 underline that, while MILP formulations give a rigorous framework, exact algorithms struggle with scalability, leading researchers to rely heavily on heuristics and metaheuristics. Together, these insights highlight that addressing the large-scale real-world challenges faced by cities operating bike-sharing systems requires more comprehensive approaches, often combining multiple methods to achieve the most effective solutions.

Table 2.1: Static Rebalancing problem classification I.

		Demand				Bikes		Stations	
Year	Authors	Det.	Stoch.	Point	Int.	Single	Mult.	Docked	Dockless
2011	Benchimol et al. [2]	✓		✓		✓		✓	
2013	Chemla, Meunier, and Calvo [4]	✓		✓		✓		✓	
	Rainer-Harbach et al. [37]	✓		✓		✓		✓	
	Raviv, Tzur, and Forma [39]	✓		✓		✓		✓	
2014	Chira et al. [7]	✓		✓		✓		✓	
	Dell'Amico et al. [9]	✓		✓		✓		✓	
	Erdoğan, Laporte, and Calvo [15]	✓		✓	✓	✓		✓	
	Ho and Szeto [22]	✓		✓		✓		✓	
2015	Erdoğan, Battarra, and Calvo [14]	✓		✓		✓		✓	
	Forma, Raviv, and Tzur [17]	✓		✓		✓		✓	
	Rainer-Harbach et al. [38]	✓		✓		✓		✓	
2016	Alvarez-Valdes et al. [1]		✓	✓			✓	✓	
	Dell'Amico et al. [10]	✓		✓		✓		✓	
	Gaspero, Rendl, and Urli [20]	✓				✓		✓	
	Kadri, Kacem, and Labadi [25]	✓			✓	✓		✓	
	Li et al. [31]	✓		✓			✓	✓	
	Szeto, Liu, and Ho [44]	✓		✓		✓		✓	
2017	Cruz et al. [8]	✓		✓		✓		✓	
	Ho and Szeto [23]	✓		✓		✓		✓	
	Pal and Zhang [35]	✓		✓		✓			✓
	Schuijbroek, Hampshire, and Van Hoeve [40]		✓	✓	✓	✓		✓	
2018	Dell'Amico et al. [11]		✓	✓		✓		✓	
	Leclaire and Couffin [29]	✓		✓		✓		✓	
	Liu, Szeto, and Ho [32]	✓		✓		✓			✓
	Szeto and Shui [45]	✓		✓		✓		✓	
2019	Lahoorpoor et al. [26]	✓		✓		✓		✓	
	Wang and Wu [48]	✓		✓			✓	✓	
2020	Du et al. [13]	✓		✓			✓		✓
2021	Li and Liu [30]	✓		✓		✓		✓	
	Ma et al. [34]		✓	✓		✓			✓
2023	Chen et al. [6]		✓	✓		✓		✓	
	Lu, Gao, and Huang [33]	✓		✓		✓			✓
2024	Yu et al. [50]	✓			✓	✓			✓

Table 2.5: Non-exact solution approaches.

Year	Authors	Type	Description
2011	Benchimol et al. [2]	A	Solve the problem in polinomial time for special cases.
2013	Chemla, Meunier, and Calvo [4]	Ma	Step 1. Branch-and-cut algorithm to solve the MILP Relaxation.
			Step 2. Tabu search algorithm initialized with the previous solution.
	Rainer-Harbach et al. [37]	M	VNS algorithm.
			VND algorithm for local improvement.
			Maximum Flow Based Method.
	Raviv, Tzur, and Forma [39]	H	Step 1. Compute the routes relaxing integrality over loading variables.
			Step 2. Fixing the routes and compute the loading values.
2014	Chira et al. [7]	M	Approach based on GAs.
			Approach based on Ant colony systems.
	Ho and Szeto [22]	H	Iterated Tabu Search Constructive algorithm.
2015	Erdoğan, Battarra, and Calvo [14]	M	Greedy Constructive algorithm.
	Forma, Raviv, and Tzur [17]	Ma	Step 1. Cluster Stations.
			Step 2. Routing decisions.
			Step 3. Solve original problem fixing previous information.
	Rainer-Harbach et al. [38]	M	Fast Greedy Constructive algorithm.
			PILOT algorithm with GRASP algorithm.
			VND to locally improve solutions.
2016	Alvarez-Valdes et al. [1]	Ma	Step 1. Calculating the target number of bicycles per station.
			Step 2. Constructing the repositioning routes for the vehicles.
	Dell'Amico et al. [10]	M	Constructive algorithm applying solution properties.
	Gaspero, Rendl, and Urli [20]	M	LNS algorithm.
	Kadri, Kacem, and Labadi [25]	M	Branch-and-bound with Genetic, Greedy Search and Nearest Neighbor.
	Li et al. [31]	M	Combined hybrid genetic algorithm in two phases:
			Step 1. Determine routing decisions.
			Step 2. Determine loading and unloading decisions.
	Szeto, Liu, and Ho [44]	M	Step 1. CRO to handle the vehicle routes.
			Step 2. Subroutine to determine the loading and unloading quantities.
2017	Cruz et al. [8]	H	Iterated Local Search based algorithm.
	Ho and Szeto [23]	M	Hybrid LNS algorithm.
			Tabu search is further applied to the most promising solutions.
	Pal and Zhang [35]	M	Hybrid Nested LNS with VND algorithm.
	Schuijbroek, Hampshire, and Van Hoeve [40]	H	Step 1. Cluster stations.
			Step 2. Single-vehicle routing problem at each cluster.
2018	Dell'Amico et al. [11]	H	Algorithms based on correlations among stochastic variables.
	Liu, Szeto, and Ho [32]	M	Enhanced version of CRO from [44].
	Szeto and Shui [45]	M	Enhanced Artificial Bee colony algorithm
2019	Lahoorpoor et al. [26]	M	Approach based on GA
2020	Du et al. [13]	M	Greedy-Genetic Algorithm
2021	Li and Liu [30]	M	Bi-level VNS algorithm
	Ma et al. [34]	M	Approach based on GA
2023	Chen et al. [6]	H	Bisection-search Algorithm.
	Lu, Gao, and Huang [33]	H	Clustering Algorithm.
2024	Yu et al. [50]	H	Two-stage robust model based on spatio-temporal networks.
			Customized column-and-constraint generation algorithm.

A: Approximation ; H: Heuristic; M: Meta-Heuristic; Ma: Math-Heuristic
GA: Genetic Algorithm; GRASP: Greedy Randomized Adaptive Search Procedures;
VND: Variable Neighborhood Descent; VNS: Variable Neighborhood Search; LNS: Large Neighborhood Search; PILOT: Preferred Iterative LOok ahead Technique; CRO: Chemical Reaction Optimization;

Bibliography II

[1] Ramon ALVAREZ-VALDES, Jose M. BELENGUER, Enrique BENAVENT, Jose D. BERMUDEZ, Facundo MUÑOZ, Enriqueta VERCHER, and Francisco VERDEJO. "Optimizing the level of service quality of a bike-sharing system". In: *Omega* 62 (2016), pp. 163–175. DOI: 10.1016/j.omega.2015.09.007.

[2] Mike BENCHIMOL, Pascal BENCHIMOL, Benoît CHAPPERT, Arnaud DE LA TAILLE, Fabien LAROCHE, Frédéric MEUNIER, and Ludovic ROBINET. "Balancing the stations of a self service "bike hire" system". In: *RAIRO-Operations Research-Recherche Opérationnelle* 45.1 (2011), pp. 37–61. DOI: 10.1051/ro/2011102.

[3] Bruno P. BRUCK, Walton P. COUTINHO, and Pedro MUNARI. "The robust bike sharing rebalancing problem: formulations and a branch-and-cut algorithm". In: *European Journal of Operational Research* 325.1 (2025), pp. 67–80. DOI: 10.1016/j.ejor.2025.02.029.

[4] Daniel CHEMLA, Frédéric MEUNIER, and Roberto Wolfler CALVO. "Bike sharing systems: solving the static rebalancing problem". In: *Discrete Optimization* 10.2 (2013), pp. 120–146. DOI: 10.1016/j.disopt.2012.11.005.

[5] Daniel CHEMLA, Frédéric MEUNIER, Thomas PRADEAU, Roberto Wolfler CALVO, and Houssame YAHIAOUI. *Self-service bike sharing systems: simulation, repositioning, pricing*. Tech. rep. 2013. URL: https://hal.science/hal-00824078/document (visited on 09/26/2025).

[6] Qingxin CHEN, Chenyi FU, Ning ZHU, Shoufeng MA, and Qiao-Chu HE. "A target-based optimization model for bike-sharing systems: from the perspective of service efficiency and equity". In: *Transportation Research Part B: Methodological* 167 (2023), pp. 235–260. DOI: 10.1016/j.trb.2022.12.002.

[7] Camelia CHIRA, Javier SEDANO, José R. VILLAR, Mónica CÁMARA, and Emilio CORCHADO. "Urban bicycles renting systems: modelling and optimization using nature-inspired search methods". In: *Neurocomputing* 135 (2014), pp. 98–106. DOI: 10.1016/j.neucom.2013.07.051.

[8] Fábio CRUZ, Anand SUBRAMANIAN, Bruno P BRUCK, and Manuel IORI. "A heuristic algorithm for a single vehicle static bike sharing rebalancing problem". In: *Computers & Operations Research* 79 (2017), pp. 19–33. DOI: 10.1016/j.cor.2016.09.025.

[9] Mauro DELL'AMICO, Eleni HADJICOSTANTINOU, Manuel IORI, and Stefano NOVELLANI. "The bike sharing rebalancing problem: mathematical formulations and benchmark instances". In: *Omega* 45 (2014), pp. 7–19. DOI: 10.1016/j.omega.2013.12.001.

[10] Mauro DELL'AMICO, Manuel IORI, Stefano NOVELLANI, and Thomas STÜTZLE. "A destroy and repair algorithm for the bike sharing rebalancing problem". In: *Computers & Operations Research* 71 (2016), pp. 149–162. DOI: 10.1016/j.cor.2016.01.011.

[11] Mauro DELL'AMICO, Manuel IORI, Stefano NOVELLANI, and Anand SUBRAMANIAN. "The bike sharing rebalancing problem with stochastic demands". In: *Transportation Research Part B: Methodological* 118 (2018), pp. 362–380. DOI: 10.1016/j.trb.2018.10.015.

[12] Paul DEMAIO. "Bike-sharing: history, impacts, models of provision, and future". In: *Journal of Public Transportation* 12.4 (2009), pp. 41–56. DOI: 10.5038/2375-0901.12.4.3.

[13] Mingyang Du, Lin Cheng, Xuefeng Li, and Fang Tang. "Static rebalancing optimization with considering the collection of malfunctioning bikes in free-floating bike sharing system". In: *Transportation Research Part E* 141 (2020), paper no. 102012. DOI: 10.1016/j.tre.2020.102012.

[14] Güneş Erdoğan, Maria Battarra, and Roberto Wolfler Calvo. "An exact algorithm for the static rebalancing problem arising in bicycle sharing systems". In: *European Journal of Operational Research* 245.3 (2015), pp. 667–679. DOI: 10.1016/j.ejor.2015.03.043.

[15] Güneş Erdoğan, Gilbert Laporte, and Roberto Wolfler Calvo. "The static bicycle relocation problem with demand intervals". In: *European Journal of Operational Research* 238.2 (2014), pp. 451–457. DOI: 10.1016/j.ejor.2014.04.013.

[16] Elliot Fishman, Simon Washington, and Narelle Haworth. "Bike share: a synthesis of the literature". In: *Transport Reviews* 33.2 (2013), pp. 148–165. DOI: 10.1080/01441647.2013.775612.

[17] Iris A. Forma, Tal Raviv, and Michal Tzur. "A 3-step math heuristic for the static repositioning problem in bike-sharing systems". In: *Transportation Research Part B: Methodological* 71 (2015), pp. 230–247. DOI: 10.1016/j.trb.2014.10.003.

[18] Christine Fricker and Nicolas Gast. "Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity". In: *Euro Journal on Transportation and Logistics* 5 (2016), pp. 261–291. DOI: 10.1007/s13676-014-0053-5.

[19] Tatsuya Fukushige, Dillon T. Fitch, and Susan Handy. "Can an incentive-based approach to rebalancing a dock-less bike-share system work? Evidence from Sacramento, California". In: *Transportation Research Part A: Policy and Practice* 163 (2022), pp. 181–194. DOI: 10.1016/j.tra.2022.07.011.

[20] Luca Di Gaspero, Andrea Rendl, and Tommaso Urli. "Balancing bike sharing systems with constraint programming". In: *Constraints* 21.2 (2016), pp. 318–348. DOI: 10.1007/s10601-015-9182-1.

[21] Zulqarnain Haider, Alexander Nikolaev, Jee Eun Kang, and Changhyun Kwon. "Inventory rebalancing through pricing in public bike sharing systems". In: *European Journal of Operational Research* 270.1 (2018), pp. 103–117. DOI: 10.1016/j.ejor.2018.02.053.

[22] Sin C. Ho and Wai Yuen Szeto. "Solving a static repositioning problem in bike-sharing systems using iterated tabu search". In: *Transportation Research Part E: Logistics and Transportation Review* 69 (2014), pp. 180–198. DOI: 10.1016/j.tre.2014.05.017.

[23] Sin C. Ho and Wai Yuen Szeto. "A hybrid large neighborhood search for the static multi-vehicle bike-repositioning problem". In: *Transportation Research Part B: Methodological* 95 (2017), pp. 340–363. DOI: 10.1016/j.trb.2016.11.003.

[24] Yu Jin, Cesar Ruiz, and Haitao Liao. "A simulation framework for optimizing bike rebalancing and maintenance in large-scale bike-sharing systems". In: *Simulation Modelling Practice and Theory* 115 (2022), paper no. 102422. DOI: 10.1016/j.simpat.2021.102422.

[25] Ahmed Abdelmoumene Kadri, Imed Kacem, and Karim Labadi. "A branch-and-bound algorithm for solving the static rebalancing problem in bicycle-sharing systems". In: *Computers & Industrial Engineering* 95 (2016), pp. 41–52. DOI: 10.1016/j.cie.2016.02.002.

[26] Bahman Lahoorpoor, Hamed Faroqi, Abolghasem Sadeghi-Niaraki, and Soo-Mi Choi. "Spatial cluster-based model for static rebalancing bike sharing problem". In: *Sustainability* 11 (2019), paper no. 3205. DOI: 10.3390/su11113205.

[27] Gilbert LAPORTE, Frédéric MEUNIER, and Roberto WOLFLER CALVO. "Shared mobility systems". In: *4or* 13 (2015), pp. 341–360. DOI: 10.1007/s10288-015-0301-z.

[28] Gilbert LAPORTE, Frédéric MEUNIER, and Roberto WOLFLER CALVO. "Shared mobility systems: an updated survey". In: *Annals of Operations Research* 271 (2018), pp. 105–126. DOI: 10.1007/s10479-018-3076-8.

[29] Patrice LECLAIRE and Florent COUFFIN. "Method for static rebalancing of a bike sharing system". In: *IFAC-PapersOnLine* 51.11 (2018), pp. 1561–1566. DOI: 10.1016/j.ifacol.2018.08.274.

[30] Yanfeng LI and Yang LIU. "The static bike rebalancing problem with optimal user incentives". In: *Transportation Research Part E: Logistics and Transportation Review* 146 (2021), p. 102216. DOI: 10.1016/j.tre.2020.102216.

[31] Yanfeng LI, Wai Yuen SZETO, Jiancheng LONG, and Chin Sum SHUI. "A multiple type bike repositioning problem". In: *Transportation Research Part B: Methodological* 90 (2016), pp. 263–278. DOI: 10.1016/j.trb.2016.05.010.

[32] Ying LIU, Wai Yuen SZETO, and Sin C. HO. "A static free-floating bike repositioning problem with multiple heterogeneous vehicles, multiple depots, and multiple visits". In: *Transportation Research Part C: Emerging Technologies* 92 (2018), pp. 208–242. DOI: 10.1016/j.trc.2018.02.008.

[33] Chen LU, Linjie GAO, and Yuqiao HUANG. "Exploring travel patterns and static rebalancing strategies for dockless bike-sharing systems from multi-source data: a framework and case study". In: *Transportation Letters* 15.4 (2023), pp. 336–349. DOI: 10.1080/19427867.2022.2051798.

[34] Guanhua MA, Bowen ZHANG, Changjing SHANG, and Qiang SHEN. "Rebalancing stochastic demands for bike-sharing networks with multi-scenario characteristics". In: *Information Sciences* 554 (2021), pp. 177–197. DOI: 10.1016/j.ins.2020.12.044.

[35] Aritra PAL and Yu ZHANG. "Free-floating bike sharing: Solving real-life large-scale static rebalancing problems". In: *Transportation Research Part C* 80 (2017), pp. 92–116. DOI: 10.1016/j.trc.2017.03.016.

[36] Samarth J. PATEL, Robin QIU, and Ashkan NEGAHBAN. "Incentive-based rebalancing of bike-sharing systems". In: *Advances in Service Science*. None. Springer Proceedings in Business and Economics. Springer, 2019, pp. 21–30. DOI: 10.1007/978-3-030-04726-9_3.

[37] Marian RAINER-HARBACH, Petrina PAPAZEK, Bin HU, and Günther R RAIDL. "Balancing bicycle sharing systems: a variable neighborhood search approach". In: *European conference on evolutionary computation in combinatorial optimization*. 2013, pp. 121–132. DOI: 10.1007/978-3-642-37198-1_11.

[38] Marian RAINER-HARBACH, Petrina PAPAZEK, Günther R RAIDL, Bin HU, and Christian KLOIMÜLLNER. "PILOT, GRASP, and VNS approaches for the static balancing of bicycle sharing systems". In: *Journal of Global Optimization* 63 (2015), pp. 597–629. DOI: 10.1007/s10898-014-0147-5.

[39] Tal RAVIV, Michal TZUR, and Iris A FORMA. "Static repositioning in a bike-sharing system: models and solution approaches". In: *EURO Journal on Transportation and Logistics* 2.3 (2013), pp. 187–229. DOI: 10.1007/s13676-012-0017-6.

[40] Jasper SCHUIJBROEK, Robert C HAMPSHIRE, and Willem-Jan VAN HOEVE. "Inventory rebalancing and vehicle routing in bike sharing systems". In: *European Journal of Operational Research* 257.3 (2017), pp. 992–1004. DOI: 10.1016/j.ejor.2016.08.029.

[41] Susan A. Shaheen, Stacey Guzman, and Hua Zhang. "Bikesharing in Europe, the Americas, and Asia: past, present, and future". In: *Transportation Research Record* 2143.1 (2010), pp. 159–167. doi: 10.3141/2143-20.

[42] Chin Sum Shui and Wai Yuen Szeto. "A review of bicycle-sharing service planning problems". In: *Transportation Research Part C: Emerging Technologies* 117 (2020), p. 102648. doi: 10.1016/j.trc.2020.102648.

[43] Shouheng Sun and Myriam Ertz. "Contribution of bike-sharing to urban resource conservation: the case of free-floating bike-sharing". In: *Journal of Cleaner Production* 280 (2021), p. 124416. doi: 10.1016/j.jclepro.2020.124416.

[44] Wai Yuen Szeto, Ying Liu, and Sin C. Ho. "Chemical reaction optimization for solving a static bike repositioning problem". In: *Transportation Research Part D: Transport and Environment* 47 (2016), pp. 104–135. doi: 10.1016/j.trd.2016.05.005.

[45] Wai Yuen Szeto and Chin Sum Shui. "Exact loading and unloading strategies for the static multi-vehicle bike repositioning problem". In: *Transportation Research Part B: Methodological* 109 (2018), pp. 176–211. doi: 10.1016/j.trb.2018.01.007.

[46] Zihao Tian, Jing Zhou, Wai Yuen Szeto, Lixin Tian, and Wenbin Zhang. "The rebalancing of bike-sharing system under flow-type task window". In: *Transportation Research Part C* 112 (2020), pp. 1–27. doi: 10.1016/j.trc.2020.01.015.

[47] Thibaut Vidal, Gilbert Laporte, and Piotr Matl. "A concise guide to existing and emerging vehicle routing problem variants". In: *European Journal of Operational Research* 286.2 (2020), pp. 401–416. doi: 10.1016/j.ejor.2019.10.010.

[48] Shijin Wang and Ruochen Wu. "The Static Rebalancing Problem in Bicycle-Sharing Systems with Unusable Bicycles". In: *2019 16th International Conference on Service Systems and Service Management (ICSSSM)*. 2019, pp. 1–6. doi: 10.1109/ICSSSM.2019.8887745.

[49] Chun-An Yang, Shih-Chieh Chen, Jian-Jhih Kuo, Yi-Hsuan Peng, Yu-Wen Chen, and Ming-Jer Tsai. "Battery swapping tour optimization problem in dockless electric bike sharing service systems with distance-aware user incentives". In: *IEEE Transactions on Services Computing* 18.4 (2025), pp. 1938–1951. doi: 10.1109/TSC.2025.3562337.

[50] Lijun Yu, Yaogeng Xu, Haochen Shi, and Zhimin Zhang. "Robust optimization model based on a static rebalancing design for bike-sharing systems affected by demand uncertainty". In: *IEEE Access* 12 (2024). doi: 10.1109/ACCESS.2024.3405643.

Chapter III

Charging Systems

Electric scooters (e-scooters) are important to urban micromobility as they provide a convenient solution for short-distance travel in cities. The growth in e-scooter usage necessitates efficient charging systems. The E-Scooter Chargers Allocation (ESCA) problem optimizes charging operations by designing routes for chargers to either collect e-scooters or swap their batteries, ensuring each e-scooter is visited once, respecting chargers' capacity limits, and minimizing total travel distance. This chapter primarily surveys recent studies on infrastructure availability, technological considerations, and the effectiveness of e-scooter charging systems, while also exploring research on e-bike charging systems for its potential relevance to e-scooter charging systems. Current unregulated systems encourage competition, violence, and inefficiencies. Efficient ESCA solutions can enhance on-time charging, reduce conflicts, and promote sustainable e-scooter operations.

Keywords: e-scooters, charging systems, micromobility, battery swapping, rebalancing, mixed integer linear programming.

III

Charging Systems for Micromobility

Hadi Alizadeh, Gebze Technical University, Türkiye
Aslıhan Gür, Gebze Technical University, Türkiye
Fatih Yetgin, Gebze Technical University, Türkiye

Electric scooters (e-scooters) and electric bikes (e-bikes) have become an integral component of urban mobility, offering a convenient solution for short-distance travel. Their fast-growing usage in cities shows the need for better charging options. These shared micromobility services use two charging methods: *collection-based charging*, where e-scooters or e-bikes are gathered at specific locations for charging, and *on-site charging*, which involves battery swapping at the micromobility vehicle's location. This chapter aims to explore the findings of recent studies that consider user preferences, infrastructure availability, technological considerations, and the effectiveness of these charging systems.

Many e-scooter/e-bike companies use freelancer chargers who compete to collect and charge these vehicles. There is no regulation on the assignment of these vehicles to chargers, and the current practice is based on a 'first come, first served' system. This increases competition, encourages violence, and leads to longer distances traveled to collect these micromobility vehicles by chargers. Efficiently allocating charging personnel to these vehicles can ensure timely recharging while also enabling higher earnings for the personnel.

In this chapter, we will review the relevant literature on charging and battery swapping systems for e-scooters and e-bikes and highlight open research directions.

3.1 Charging systems for e-scooters

The E-Scooter Chargers Allocation (ESCA) problem aims to optimize e-scooter charging operations within a city [3]. Specifically, it involves finding optimal tours for all chargers to pick up e-scooters in the form of routes, such that each route contains one charger, each e-scooter is visited only once by the set of routes, and the total travel distance for chargers is minimized. In this scenario, chargers move from certain starting points such as charging stations or their homes to collect assigned e-scooters. Each charger has a maximum capacity limit that must not be exceeded during the collection process.

Masoud et al. [3] addressed the challenges of freelance chargers competing to collect and charge dockless e-scooters, which leads to inefficiencies and potential conflicts. They formulated the problem as a mixed-integer linear programming (MILP) model to minimize the average distance traveled by chargers. Additionally, due to MILP's computational complexity for large-scale instances, the authors adapted two heuristic algorithms—the College Admission Algorithm (ACA) and the Black Hole Optimizer (BHO)—as alternative approaches to find near-optimal solutions more efficiently, with ACA performing closer to optimal MILP results than BHO and being recommended for large instances. Later, Masoud et al. [4] refined the ESCA as an NP-complete generalized assignment problem, reiterating the MILP formulation and heuristic approaches ACA and BHO. Experimental results on small and medium cases shows ACA's superiority in balancing optimality and computational efficiency, recommending it for real-world large-scale instances.

Extending this study, in [5], Masoud et al. advanced the ESCA framework by focusing on static allocation scenarios, where e-scooter and charger positions are fixed resembling an urban environment like Brisbane, Australia, where chargers use private vehicles to collect e-scooters. They developed an MILP model for optimal tours assigning e-scooters to chargers, minimizing total costs including travel and recruitment, and adapted a simulated annealing (SA) metaheuristic to approximate near-optimal solutions to efficiently handle large-scale instances.

With the introduction of e-scooters with swappable batteries, new charging systems have emerged, moving beyond the collection-based charging of e-scooters with fixed batteries, while some research studies also consider concepts like rebalancing alongside charging to optimize operational efficiency. Rebalancing simply means moving e-scooters from areas with too many vehicles to places where more people need them. The objective is to ensure that e-scooters are available to users whenever and wherever needed, while also maintaining sufficient battery charge. Due to high costs and inefficiencies, many e-scooter companies have transitioned from crowd-based to operator-based overnight charging and rebalancing. However, it remains a significant operational challenge to ensure that all e-scooters are fully charged and placed in a reliable yet cost-effective manner.

The authors in [6] proposed an alternative to conventional depot-based charging, which is an on-board charging strategy that allows e-scooters to recharge while being transported during relocation tours. This approach improves operational flexibility, reduces downtime, and eliminates the need for stationary charging stops. They formulated the problem as an MILP under a multi-commodity inventory routing framework, treating scooters of different state-of-charge levels as separate commodities. To ensure scalability, they introduced a discrete-continuous hybrid model that integrates zone-level continuous approximations with discrete routing decisions. The computational results, including a case study in Washington, D.C., demonstrate that the proposed approach provides high-quality solutions with significantly reduced computation time.

Çakır et al. [1] addressed the operational planning for collecting low-charge e-scooters and battery replacement by proposing a time-constrained MILP model based on the milk-run logistics approach. The objective is to minimize the total cost, including travel distances, penalties for violating location-specific time windows, and penalties for unvisited demand points. In the proposed system, identical trucks depart from a central hub to perform either on-site battery replacements or collect e-scooters with fixed batteries to transport them back to the depot. Each location has predefined parameters such as time windows, product volumes, and loading durations. The model incorporates routing decisions, arrival times, and capacity constraints to ensure efficient vehicle utilization. A numerical case study based on an e-scooter sharing operator in Istanbul demonstrates the applicability of the model, revealing the impact of time window constraints on route configuration and total system cost. The results highlight the trade-off between operational efficiency and service punctuality, emphasizing the need for strategic planning in micromobility logistics under temporal and spatial constraints.

The work in [9] introduced energy-informed demand, a concept that integrates battery levels and energy consumption into predicting user demand for shared electric vehicles like e-scooters and e-bikes. Their framework, RECOMMEND, optimizes vehicle rebalancing and charging by using real-time data on usage, battery status, and demand patterns to make coordinated decisions, improving on traditional methods that handle these tasks separately. The framework was tested using real-world data from a micromobility operator, with experiments demonstrating its potential to enhance service reliability and operational efficiency in urban environments. By addressing the interplay between vehicle placement and energy needs, RECOMMEND ensures vehicles are both available and sufficiently charged to meet user needs.

Lee et al. [2] presented a comprehensive mixed-integer programming (MIP) model for optimizing battery swapping, vehicle rebalancing, and staff routing in free-floating electric scooter (FES) systems. Unlike station-based systems, FES requires addressing spatially dispersed demand and vehicle states under operational constraints. The proposed model maximizes profit by balancing expected revenue—derived from meeting regional demand with sufficiently charged scooters—with operational costs, including battery-swapping, loading, and routing times. The formulation captures detailed system dynamics by modeling scooter-level decisions, staff time budgets, vehicle capacities, and endogenous drop-off locations, while explicitly allowing battery swapping even without vehicle relocation. The study incorporates various solution components such as MST-based clustering and vehicle routing logic and evaluates them through illustrative examples. Notably, it assumes staff do not return to the depot mid-route and imposes no explicit limit on the number of swappable batteries carried. The framework provides a rigorous decision-support tool for operators managing FES logistics under real-world constraints.

In [7], Osorio et al. explored how to manage overnight charging and repositioning of shared e-scooters in a system using charging hubs. In the considered scenario, a vehicle starts from a central depot, collects low-battery scooters from various locations, and takes them to hubs with limited space for overnight charging. The next day, it redistributes the charged scooters to the required points before returning to the depot. Each scooter has a different battery level, which improves during charging. The researchers first use an MILP to plan the vehicle's route, decide pick-ups and drop-offs, and track battery levels all at once. However, this model is slow for large systems, so they develop a faster hybrid model that groups locations into zones and estimates routes, achieving near-optimal results with less computing time. Using a case study implemented in Washington, D.C. with 392 locations and 4 hubs, they find that having multiple smaller charging hubs with several vehicles is much more efficient than one central hub. The study highlights the value of spread-out charging hubs and suggests future research on adding battery-swapping tech, real-time route planning, and handling unpredictable demand in e-scooter systems.

Table 3.1 consolidates the main insights and approaches from the reviewed studies, offering a concise summary of the shared e-scooter charging systems.

Paper	Problem	Model	Objective	Method / Algorithm	Case Study
Masoud et al. [3]	Optimal assignment of scooters to chargers	MILP	Minimize distance + penalty for new chargers	MILP, ACA (College Admission), BHO	—
Masoud et al. [4]	Assignment of scooters to freelance chargers	MILP	Minimize average distance	MILP (small), ACA, BHO	Brisbane, Australia
Osorio et al. [6]	Optimal rebalancing + on-board charging	MIP	Maximize net benefit (revenues – costs)	Hybrid discrete–continuous approx. + MIP	Washington, DC (USA)
Masoud et al. [5]	Assignment of scooters to freelance chargers	MILP	Minimize distance + charger deployment cost	Simulated Annealing (SA)	Brisbane, Australia
Tan et al. [9]	Joint rebalancing and charging with energy-informed demand	STMIP	Maximize net revenue (trip income – charging & routing costs)	Multi-Agent RL + demand prediction + STMIP	China (operator dataset)
Çakır et al. [1]	Milk-run with time windows for battery collection/replacement	MILP	Minimize total cost (routing + time)	VRP with time windows (milk-run)	Istanbul, Türkiye
Lee et al. [2]	Battery swapping + vehicle rebalancing + staff routing (free-floating)	MIP	Maximize profit (served demand – operational cost)	Clustered Iterative Construction Approach (CICA)	Seoul, South Korea
Osorio et al. [7]	E-scooter collection, hub charging, redistribution	MIP	Minimize operational cost (routing + charging)	Hybrid discrete–continuous model, direct MIP	Washington DC

Table 3.1: Comparative Analysis of Studies on E-Scooter Charging Systems

3.2 Charging systems for e-bikes

Electric bike-sharing systems (EBS) are experiencing swift growth as a convenient mode of short-distance travel. This rapid expansion highlights the necessity for efficient charging infrastructure to facilitate their widespread adoption. Although e-bike charging systems share similarities with e-scooter charging systems, such as the use of battery-swapping mechanisms, they differ in their infrastructure requirements and power demands.

E-bike systems face critical challenges in ensuring sufficient battery availability at stations. Since there is no charging infrastructure at docking points, operators must use trucks to swap depleted batteries for fully charged ones from a central warehouse. The main challenge is that trucks often need to perform multiple replenishment trips, exacerbated by the variable and unpredictable battery demand at stations due to fluctuating user behaviors.

Yang et al. [11] modeled this operational problem as a vehicle routing problem with intermediate stops and soft time windows. They proposed a hybrid solution framework that combines a dynamic planning strategy, in which the planning horizon is divided and updated iteratively according to real-time demand information, with a multiple neighborhood search algorithm for efficient route optimization. Benchmark tests and a real-world case study in Shanghai demonstrate that this approach significantly improves system performance as it reduces transportation costs by approximately 11.5% compared to static planning while enhancing service quality and robustness.

Free-floating bike-sharing systems also present significant challenges regarding rebalancing and maintenance. China has introduced a new generation of electric bike-sharing systems (EBSS), which combine the flexibility of free-floating services with the control of electric fenced parking zones [12]. In these systems, users must return bikes to designated zones. Charging is managed through a central depot and a network of street-side battery cabinets. Two groups of staff support these operations: swappers, who replace depleted batteries, and relocators, who redistribute bikes to balance supply and demand.

Zhou et al. [12] developed a Markov chain-based framework for EBSS that considers bike inventory and battery levels at stations and cabinets. The authors introduced a one-step Markovian strategy for adaptive short-term operations and a rolling-horizon strategy for long-term planning that is globally optimal. Using real operational data from Xiamen, they conducted numerical experiments which demonstrate that these strategies substantially outperform existing rule-based practices. Together, they lead to more than a 20% improvement in profit, enhanced service quality, and greater operational sustainability.

Shao et al. [8] addressed the large-scale operational problem of battery swapping in city-wide EBSS by modeling the problem as an MILP formulation of a multi-depot vehicle routing problem with intermediate facilities and soft time windows. The system assumes that trucks leave from distributed depots, repeatedly visit warehouses to collect fully charged batteries, and swap depleted ones at e-bike stations within flexible service intervals. The authors suggest a three-stage heuristic to overcome the computational difficulty of solving the MILP directly for city-scale instances: (i) capacitated k-medoids clustering to partition the city into service regions anchored by warehouses, (ii) fan-shaped clustering of stations within each region to reduce problem size and align with driver practices, and (iii) a two-layer Adaptive Large Neighborhood Search (ALNS) that iteratively improves intra- and inter-cluster routing. Tested on Hefei's EBSS with 4,500 stations, the framework reduces costs by about 25% compared to grid-based clustering, while delivering near-optimal quality with only 0.5% of the computation time required by direct ALNS.

The EBSS battery swapping problem is precisely described by the suggested MILP formulation at the modeling level [8]. The proposed MILP formulation minimizes total travel distance, vehicle operating costs, and penalty costs for violating soft time windows where early swaps waste charging resources and late swaps result in lost demand. Sets and parameters cover e-bike stations, depots, warehouses, vehicle capacities, historical demand, and service time windows. Decision variables include binary arc selections, warehouse visits, and depot assignments, alongside continuous variables for arrival times, onboard battery loads, and penalty linearizations. Constraints guarantee feasibility by enforcing capacity, time, and visit requirements. Importantly, the MILP is solvable by commercial solvers only for small instances; for large-scale, city-wide problems, the three-stage heuristic serves as a scalable solution method complementing the MILP, ensuring practical applicability while maintaining solution quality.

In their research work, Yang et al. [10] studied the battery swapping problem in dockless e-bike sharing systems, focusing on how to reduce both truck routing costs and user incentive costs. The authors introduce algorithms that combine vehicle routing optimization with a distance-aware incentive mechanism, encouraging users to relocate bikes to more convenient parking locations. By integrating these two aspects, their approach reduces the number of truck stops needed for battery swapping while keeping incentives at a reasonable level. Simulation results with real-world data show that the proposed methods outperform classical routing heuristics, highlighting the value of incentive-assisted routing strategies for efficient e-bike battery swapping operations.

3.3 Conclusions

This survey highlights the crucial role of charging systems in supporting urban micromobility by focusing on e-scooters and e-bikes and by providing efficient and sustainable solutions for short-distance travel. The E-Scooter Chargers Allocation (ESCA) problem focuses on creating charger routes to collect or swap e-scooter batteries, ensuring each scooter is serviced once, following capacity limits, and reducing travel distance. Recent studies show challenges like limited charging stations and issues in unregulated systems, which often lead to competition and inefficiencies. By reviewing infrastructure, technology, and performance, this section points to ways to improve timely charging and support sustainable e-scooter use, helping expand urban micromobility networks.

The section on e-bike charging systems adds value by exploring their potential connection to e-scooter charging. Research on e-bikes shows unique challenges, such as managing batteries in station-based and free-floating systems, driven by changing user patterns and spread-out locations. While e-bike systems share features with e-scooters, like battery-swapping, they differ in their station needs and power use, offering useful solutions for micromobility. This section shows the benefit of studying related systems to address common issues in urban transport.

In summary, improving charging systems for e-scooters, with insights from e-bike research, is key to strengthening urban micromobility. Effective ESCA solutions can reduce inefficiencies, limit conflicts, and promote eco-friendly transport networks. Future work should focus on using smart charging systems and predicting demand to tackle the challenges of e-scooter and e-bike systems.

Bibliography III

[1] Esra Çakir, Özgür İzcan, and Emre Demircioğlu. "Milk-run optimization under time windows for collection and replacement of low-charge e-scooters". In: *IEEE Forum on Research and Technologies for Society and Industry Innovation*. IEEE. 2024, pp. 590–594. doi: 10.1109/RTSI61910.2024.10761872.

[2] Gaeun Lee, Jun Soo Lee, and Kun Soo Park. "Battery swapping, vehicle rebalancing, and staff routing for electric scooter sharing systems". In: *Transportation Research Part E: Logistics and Transportation Review* 186 (2024), p. 103540. doi: 10.1016/j.tre.2024.103540.

[3] Mahmoud Masoud, Mohammed Elhenawy, Mohammed H Almannaa, Shi Qiang Liu, Sebastian Glaser, and Andry Rakotonirainy. "Optimal assignment of e-scooter to chargers". In: *IEEE Intelligent Transportation Systems Conference*. IEEE. 2019, pp. 4204–4209. doi: 10.1109/ITSC.2019.8917255.

[4] Mahmoud Masoud, Mohammed Elhenawy, Mohammed H Almannaa, Shi Qiang Liu, Sebastien Glaser, and Andry Rakotonirainy. "Heuristic approaches to solve e-scooter assignment problem". In: *IEEE Access* 7 (2019), pp. 175093–175105. doi: 10.1109/ACCESS.2019.2957303.

[5] Mahmoud Masoud, Mohammed Elhenawy, Shi Qiang Liu, Mohammed Almannaa, Sebastien Glaser, and Wael Alhajyaseen. "A simulated annealing for optimizing assignment of e-scooters to freelance chargers". In: *Sustainability* 15.3 (2023), p. 1869. doi: 10.3390/su15031869.

[6] Jesus Osorio, Chao Lei, and Yanfeng Ouyang. "Optimal rebalancing and on-board charging of shared electric scooters". In: *Transportation Research Part B: Methodological* 147 (2021), pp. 197–219. doi: 10.1016/j.trb.2021.03.009.

[7] Jesus Osorio and Yanfeng Ouyang. "Optimal rebalancing and charging of shared electric scooters using charging hubs". In: *Transportation Research Part E: Logistics and Transportation Review* 196 (2025), paper no. 104018. doi: 10.1016/j.tre.2025.104018.

[8] Siyu Shao, Deyi Li, Yaoming Zhou, and Jiuh-Biing Sheu. "Optimizing battery swapping for city-scale e-bike sharing systems: A three-stage spatial – temporal cluster-based approach". In: *Transportation Research Part E: Logistics and Transportation Review* 198 (2025), paper no. 104145. doi: 10.1016/j.tre.2025.104145.

[9] Heng Tan, Yukun Yuan, Shuxin Zhong, and Yu Yang. "Joint rebalancing and charging for shared electric micromobility vehicles with energy-informed demand". In: *Proceedings of the 32nd ACM International Conference on Information and Knowledge Management*. 2023, pp. 2392–2401. doi: 10.1145/3583780.3614942.

[10] Chun-An Yang, Shih-Chieh Chen, Jian-Jhih Kuo, Yi-Hsuan Peng, Yu-Wen Chen, and Ming-Jer Tsai. "Battery swapping tour optimization problem in dockless electric bike sharing service systems with distance-aware user incentives". In: *IEEE Transactions on Services Computing* 18.4 (2025), pp. 1938–1951. doi: 10.1109/TSC.2025.3562337.

[11] Yi Yang, Yaoming Zhou, Tanmoy Kundu, and Suxiu Xu. "Dynamic vehicle routing for battery swapping in an electric bike-sharing system". In: *IEEE International Conference on Industrial Engineering and Engineering Management*. IEEE. 2021, pp. 678–682. doi: 10.1109/IEEM50564.2021.9672818.

[12] Yaoming Zhou, Zeyu Lin, Rui Guan, and Jiuh-Biing Sheu. "Dynamic battery swapping and rebalancing strategies for e-bike sharing systems". In: *Transportation Research Part B: Methodological* 177 (2023), paper no. 102820. DOI: https://doi.org/10.1016/j.trb.2023.102820.

Chapter IV

Public Transportation

Planning in public transportation requires coordinated decisions across multiple levels, from designing networks to managing daily operations. Strategic planning determines system capacity through choices on depots, fleets, and routes. Tactical planning aligns supply with demand by setting service frequencies and timetables, while operational planning allocates vehicles and crews to scheduled trips. The associated optimization tasks — line planning, timetabling, vehicle and crew scheduling, and rostering — are computationally demanding, particularly when considered together. To address these challenges, both sequential and integrated solution approaches have been studied. This review surveys key problem classes and also includes graph-theoretic concepts that support transportation and traffic control analysis.

Keywords: Crew scheduling, vehicle scheduling, rostering, public transportation, timetabling, traffic control.

IV

Public Transportation and Traffic Modeling

Taner Bilgiç, Boğaziçi University, Türkiye
Balázs Dávid, University of Primorska, Slovenia
Tınaz Ekim, Boğaziçi University, Türkiye
Didem Gözüpek, Gebze Technical University, Türkiye
Sibel Özkan, Gebze Technical University, Türkiye
Nour Elhouda Tellache, University of Fribourg, Switzerland
Bilge Varol, Boğaziçi University, Türkisch-Deutsche Universität, Türkiye

Public transport systems are a large and complex hierarchy of interconnected planning problems, where higher-level decisions affect the set of feasible solutions at lower levels. Optimization problems in public transportation are often categorized into three different planning phases: *strategic*, *tactical*, and *operational* [47].

The **strategic** level is responsible for long-term structural decisions that affect the size and infrastructure of the entire system, such as the design of the transit network, as well as decisions about depot locations and the size and heterogeneity of the fleet. At the **tactical** level, medium-term choices responsible for the provided service are made: the planning of lines and their frequencies, ultimately resulting in timetables. These are the decisions that connect the physical transport network with potential passenger flows and determine the service tasks to be executed in the scheduling phase. The final **operational** level is responsible for the creation of the actual day-to-day schedules through the assignment of vehicles and drivers to the tasks arising in the system.

The main optimization problems distinguished within this hierarchy are usually the following:

- **Line planning:** determines the routes of public transport lines and the places of stops on these.
- **Timetabling:** defines departure and arrival times of the service trips along the lines determined previously.
- **Vehicle scheduling:** assigns timetabled trips to individual vehicles in the fleet of the company, depending on constraints such as fleet size and vehicle types, depot capacities and other operational regulations.
- **Crew scheduling:** constructs feasible daily shifts by grouping vehicle duties into assignments that satisfy worker regulations.
- **Crew rostering:** assigns daily shifts to actual employees over a planning horizon, taking into account labor laws and contractual regulations.

Each of the above subproblems is computationally hard even when considered alone, and their interdependence further increases the complexity of solving multiple subseqent stages together efficiently. As a result, the literature has focused both on the development of sequential algorithms for solving subproblems individually and on integrated approaches that simultaneously address multiple levels of decision making.

This review will follow the outline above to present the most important general concepts of public transport and introduce other problem variations that might arise in the daily scheduling process of public transport companies.

4.1

Vehicle scheduling

Block time scheduling, also known as the **vehicle scheduling problem (VSP)** is a pivotal optimization challenge within public transportation systems. Its core objective is the efficient **assignment of vehicles to a predetermined timetable of trips** by minimizing operational expenditures, particularly those associated with vehicle fleet size and non-revenue-generating activities such as layovers and deadhead trips, all while upholding service reliability.

Timetabled trips represent the individual service activities that the vehicles have to cover, transferring passengers between two terminal stations - also visiting the corresponding intermediate stations - in a given time interval. Timetabled trips are defined by four important characteristics: starting and ending location, and departure and arrival time. A vehicle block is a set of pairwise compatible timetabled trips, where **compatibility** of two trips means that they can be serviced by the same vehicle without any time conflict.

Two sequential timetabled trips do not necessarily have to end and start at the same location, a vehicle can change locations without carrying passengers through travel activities called **deadhead trips**.

The prominent formulations are as follows:

- Network Flow Models,
- Set Partitioning/Covering Models.

Both types of formulations need a representation of the underlying network structure. Connection networks or time-space networks are utilized for this purpose.

The single-depot vehicle scheduling problem (SD-VSP) is introduced in [9]. Multiple-depot VSP (MD-VSP) has attracted more attention since the 1990s and has been shown to be an NP-Hard problem in [7].

One of the early approaches to the solution of the problem proposes a multi-commodity flow formulation of the MD-VSP and develops a column generation technique based on Lagrangian pricing with an application to three German public transportation companies [54]. Specialized branch and cut approaches are also proposed for its solution [39].

In this section, we consider **integrated formulations** of VSP and Crew Scheduling. Integrated formulations in which vehicle scheduling is performed simultaneously with crew scheduling appeared first in the airline industry (see, e.g., [6]). The sequential workflow first builds vehicle blocks to minimize the fleet and deadheads. Then, it tries to fit crew duties to those blocks, considering breaks, spread, and costs. This approach can miss important couplings between vehicle and crew planning. An integrated model can trade small vehicle changes for big improvements in crew feasibility and cost.

In the context of public bus planning, flow-based formulations are proposed in time space networks, and columns-based solutions have been shown to be effective for the integrated vehicle and crew scheduling problem [46, 49, 70]. Set partitioning/covering formulations are also available and can solve larger instances of the problem [59].

Recent approaches extend beyond traditional deterministic assumptions by incorporating stochastic and data-driven models, which account for real-world uncertainties and delays to enhance adherence and robustness of the schedule [3, 68]. Applications to electric buses are considered where the underlying vehicle scheduling problem has to explicitly consider recharging batteries [31, 65].

4.2

Crew scheduling

In the literature, the crew scheduling problem has been studied in various transportation domains such as airlines, rails, and urban bus systems. Each of these domains brings its own operational, regulatory, and organizational challenges, which in turn shape the extensions and approaches developed to address them. In the context of public transportation, researchers have explored many different directions, such as incorporating labor regulations and union agreements, enforcing time-related restrictions, considering driver preferences, promoting fairness and workload balance, linking crew scheduling with vehicle scheduling, and designing robust solutions that can withstand fluctuations in demand or unexpected disruptions [3, 12, 76, 84, 85].

Several survey studies have offered valuable insights into the crew scheduling problem across different transportation domains. One of the most influential works, Ernst et al. [18], provided a detailed classification of personnel scheduling and rostering methods, laying the foundation for subsequent research. Desaulniers et al. [17] extended this perspective by situating public transit crew scheduling within the broader framework of transportation operations. Building on this, Vera et al. [78] highlighted the close relationship between crew scheduling and rostering, emphasizing the benefits of tackling these problems in an integrated manner. More recently, Mertens et al. [58] presented a state-of-the-art overview of optimization techniques for resource scheduling in public transportation, bringing together advances in exact, heuristic, and integrated approaches. Collectively, these studies not only map the evolution of the field but also provide the conceptual and methodological basis for more specialized research on bus crew scheduling. This section focuses on the bus crew scheduling problem, with particular emphasis on its variants and the different solution methodologies proposed in the literature.

Öztop et al. [63] studied a real-life crew scheduling problem in public bus transportation, where drivers face spread time and working time limits, and different crew types are needed due to vehicle-specific requirements. They developed a binary programming model for the Tactical Fixed Job Scheduling Problem that determines the optimal number of crew members of different types at minimum cost, enhanced with an iterative valid inequality generation scheme that proved effective on instances of up to 120 tasks. Lourenço et al. [56] proposed multi-objective metaheuristics for bus-driver scheduling, combining tabu search and genetic algorithms to handle conflicting goals such as cost and service quality. This method was validated with real-world data and implemented in the GIST Planning Transportation Systems and demonstrated effectiveness in guiding efficient search and improving solution quality for crew scheduling problems. Andrade-Michel et al. [4] addressed the Flexible Vehicle and Crew Scheduling Problem in urban bus transport, incorporating vehicle characteristics, driver qualifications, and strict labor regulations such as duty length limits, mandatory breaks, and restricted driving hours. They developed a mixed-integer linear programming model and a variable neighborhood search algorithm. They demonstrated its efficiency through extensive computational experiments on large-scale instances. Pardo-Pena et al. [64] introduced a GRASP-based approach for the Bus Crew Scheduling Problem, aimed to balance cost minimization with driver satisfaction by incorporating constraints such as work blocks, rest days, and prohibited shift sequences. Tested on both classical instances and real company data, the method — enhanced with a satisfaction function — demonstrates improved solution quality, computational efficiency, and positive impacts on the working environment. Ma et al. [57] addressed fairness in the Crew Scheduling Problem for Bus Drivers by developing a hybrid ant-colony optimization algorithm that integrated a Gamma heuristic and refined selection rules. Using real data from Beijing Public Transport Holdings, the model demonstrated efficiency on large-scale instances while showing that incorporating fairness costs significantly improves balance in working and idle times without sacrificing solution quality. Wang et al. [80] developed a bi-level, multi-objective programming model for the collaborative scheduling of vehicles and drivers in mixed fleets of conventional and electric buses, addressing both operational costs and carbon emissions. By solving the problem with an improved multi-objective particle swarm algorithm, the study demonstrated that integrating vehicle charging schemes with crew scheduling not only reduces costs and emissions but also improves driver–bus specificity and compliance with labor constraints. Kang et al. [48] formulated integer linear programming models for bus driver scheduling, extending them with a derandomizing approach to handle uncertainties in bus travel times. Through a case study on Singapore's bus route #95 and sensitivity analyses on fleet size and driver workload, the models demonstrate practical applicability, while Lagrangian bounds provide insights into solution quality and managerial decision-making.

Taking crew preferences into account when building schedules has been studied primarily in the airline industry. The systems developed for this purpose can generally be classified into three categories. In the bid-line system, anonymous schedules are first constructed — where various objectives can be considered depending on the application — and crew members then bid on these schedules, which are subsequently assigned according to seniority [11]. In the rostering system, personalized schedules are computed while taking into account preassigned activities such as vacations and training periods, and optimizing an objective function, which may include factors such as cost, covered pairings, or overall crew satisfaction [27]. In the preferential bidding system, the crew members express their preferences for various activities — including pairings, preassignments, and rest periods — through bids. The system then constructs schedules that maximize crew members' scores in a lexicographic order, with the most senior crew served first, followed by the next most senior, and so on. The score of a schedule for a crew member is determined based on their submitted bids [73].

Table 4.1 summarizes the literature on crew scheduling for buses.

Paper	**Problem Focus**	**Objectives**	**Methods**	**Real Data?**	**Special Constraints/Features**	**Scale/Validation**
Lourenço et al. (2001) [56]	Bus-driver scheduling (multi-objective)	Cost + service quality	Hybrid tabu search + genetic algorithm	Yes (GIST implementation)	Conflicting goals; guides efficient search	Validated with real-world data
Öztop et al. (2017) [63]	Tactical Fixed Job Scheduling (bus drivers)	Minimize total cost; decide counts of driver types	Binary programming + iterative valid inequality generation	Yes	Spread-time and working-time limits; vehicle-specific crew types	Effective up to 120 tasks
Ma et al. (2017) [57]	Fairness in bus-driver scheduling	Fairness cost + schedule quality	Hybrid Ant Colony Optimization + Gamma heuristic	Yes (Beijing Public Transport Holdings)	Balance of working/idle times; fairness metrics	Efficient on large-scale instances
Kang et al. (2020) [48]	Bus-driver scheduling under uncertainty	Feasibility with cost/workload considerations	ILP + derandomizing approach; Lagrangian bounds	Yes (Singapore Route #95)	Travel-time uncertainty; sensitivity to fleet size and workload	Case study + sensitivity analysis
Andrade-Michel et al. (2021) [4]	Flexible Vehicle & Crew Scheduling (urban buses)	Feasible, low-cost schedules	MILP + Variable Neighborhood Search (VNS)	Extensive experiments	Vehicle characteristics; driver qualifications; duty length limits; mandatory breaks; driving-hour restrictions	Large-scale instances
Wang et al. (2022) [80]	Collaborative vehicle–driver scheduling (diesel + EV)	Cost + carbon emissions (bi-level, multi-objective)	Bi-level multi-objective model + improved MOPSO	Experimental evaluation	Charging integration; driver–bus specificity; labor constraints	Reduced cost/emissions demonstrated
Pardo-Peña et al. (2023) [64]	Bus crew scheduling with satisfaction	Balance cost and driver satisfaction	GRASP + satisfaction function	Yes (classical + company data)	Work blocks; rest days; prohibited shift sequences	Improved quality and speed reported

Table 4.1: Summary of bus crew scheduling literature.

4.3

Maintenance planning

Maintenance planning is paramount to the operational integrity, safety, and economic viability of public transportation systems. Historically, maintenance approaches have progressed from reactive interventions to scheduled preventive measures. However, contemporary demands for enhanced reliability, efficiency, and cost optimization are driving a significant paradigm shift towards predictive maintenance (PdM). This advanced strategy leverages real-time data, the Internet of Things (IoT), and analytical techniques, particularly from machine learning (ML) to forecast equipment failures and optimize maintenance schedules.

In this section, we give a brief account of maintenance planning in public transportation systems with a focus on three aspects of the problem:

- Assigning Preventive Maintenance activities
- Emphasizing Predictive Maintenance,
- Connections to Fleet Planning.

The sustained maintenance and ongoing operation of existing infrastructure is already a challenging problem given the widespread aging of much of the current transportation infrastructure, which requires a concentrated effort to maximize the performance and extend the lifespan of existing assets. Furthermore, introduction of new technologies in vehicles (electric battery powered, driverless modes etc.) adds layers to these challenges.

Maintenance directly influences key operational metrics, including overall operational costs and, crucially, the level of passenger satisfaction. Public expectations for transportation services are increasingly stringent, prioritizing not only reliability and frequency, but also broad availability and accessibility.

A critical aspect of maintenance in transportation systems is its effect on fleet planning. The fleet planning optimization models usually assume that the resources (vehicles, roads, trains, railways etc.) are always available, never fail, or there are ample redundancy such that failure is negligible. However, particularly with aging assets and infrastructure, the validity of this assumption is in question. When an asset becomes unavailable due to failure, all public transportation plans are significantly affected.

4.3.1 Preventive maintenance

Preventive maintenance is a proactive planning strategy that performs regularly scheduled activities such as inspections, cleaning, and repairs to prevent vehicle failure and prolong their expected lifespan. Due to its nature, optimizing preventive maintenance activities predominantly involves planning over a longer horizon.

Preventive maintenance activities are mainly discussed in literature related to rail transport. Borndörfer et al. [10] present a hypergraph-based MIP model for rolling stock rotation planning, where vehicle composition and maintenance constraints are also considered. Use cases are presented on the scenarios of Deutsche Bahn.

Giacco et al. [30] also integrate maintenance into the rolling stock circulation problem. Their MILP model inserts the maintenance activities into a pre-determined cyclic timetable and assignment. Giacco et al. [29] also proposed a sequential method that first creates rolling stock rosters and then assigns maintenance activities to them. The method is presented on small scenarios from the Italian railway company Trenitalia.

A MIP model and a hybrid heuristic are given by [52] for rolling stock assignment. Optimization of maintenance activities is performed on a daily basis, and a sequential rolling-horizon approach is considered for assignments over a longer horizon.

In public bus transport, Haghani and Shahafi [41] give multiple mathematical models for the insertion of different maintenance activities into an existing bus assignment over a longer horizon.

Dávid and Krész [16] consider a schedule assignment model, where daily blocks are assigned to actual buses over a planning horizon, taking into consideration depot compatibility, daily parking, and preventive maintenance activities. They give a state-expanded multi-commodity flow MILP formulation for the problem.

He et al. [42] formulate the MILP model over a day-to-day rolling planning horizon for preventive and predictive maintenance. They employ a two-stage decomposition method, where bus maintenance is scheduled on a daily basis first, then the vehicle schedules and bus maintenance activities are optimized within each day.

4.3.2 Predictive maintenance

Predictive Maintenance (PdM) emerged as an active field of research in recent years. Since it is fundamentally a data-driven approach that harnesses statistical analysis and Machine Learning (ML) models, availability of such data enabled novel approaches to maintenance planning. With higher granularity of the available data, we can model system behavior, identify degradation trends, and accurately forecast potential failures, which all contribute to enhancing overall system reliability.

Predictive maintenance is usually implemented as condition-based maintenance using data analysis to predict failures. As a result, it is expected to reduce downtime; optimize maintenance costs; improve system reliability; enhance safety [14]. The data infrastructure is critical and requires significant investment in sensors.

The stochastic nature of breakdowns is usually ignored in the public transportation planning literature and is modeled independently. Gerum et al. [55] consider both railway problems by proposing a new approach to predict rail and geometry defects that is based on easy-to-obtain data. They integrate prediction with inspection and maintenance scheduling activities. Their prediction methodology is developed to control the underestimation of defects. Then, based on a discounted Markov decision process model that uses these predictions, they determine an optimal inspection and maintenance scheduling policy. This approach has the potential to be linked to the vehicle scheduling problem in further research.

Another typical example of predictive maintenance is made possible by the availability of the MetroPT dataset [77]. This dataset enables testing different models for the prediction of failures of a critical component of Porto Metro trains (air control unit). The challenge is to predict failure without false positives and send a warning early enough so that service can continue without interruption. Although prediction methods can be tested on this dataset, it is not straightforward how to use these predictions in maintenance planning.

4.4 Graph theoretic approaches

4.4.1 Reload cost concept

The literature includes purely graph-theoretic problems that arise from applications in public transportation. One prominent example is the concept of *reload costs*. In this subsection, we will review related work on the reload cost concept.

Edge-colored graphs provide a natural framework for modeling optimization problems across diverse domains, including bioinformatics, communication networks, and transportation systems. In this context, the reload cost is defined as the expense incurred when traversing a vertex through two incident edges, where the cost is uniquely determined by the colors of these edges.

Beyond communication and energy distribution networks, the reload cost concept finds extensive applications in transportation. A notable example arises in multi-modal cargo transportation, where transfers between different modes of transport incur significant (un)loading costs at intermediate transfer points [24].

The concept of reload cost was first introduced by Wirth and Steffan [83], who studied the problem of finding a spanning tree with the minimum diameter under reload costs. Since then, a wide array of related optimization problems have been explored. These include identifying paths, trails, or walks with the minimum reload cost between specified vertices [33]; numerous path, tour, and flow problems [2]; the minimum changeover cost arborescence problem [25, 34, 35, 37]; the problem of finding a cycle cover with minimum reload costs [26]; structural and algorithmic results on minimum reload costs in complete graphs with equitable 2-edge colorings [13]; the problems of finding edge-colorings that minimize the total reload cost [36]; and constructing spanning trees that minimize the overall reload costs across all vertex pairs [28]. Furthermore, Baste et al. [5] investigated graph factors with reload costs, deriving several complexity results and identifying special cases that can be solved in polynomial time.

The concept of reload cost has been further extended to shortest-path problems with quadratic objective functions. Fischer et al. [22] examined the quadratic traveling salesman problem, where a cost is assigned to every sequence of three consecutive nodes, and analyzed its polyhedral structure. Rostami et al. [66] explored the computational complexity of the quadratic shortest path problem and proposed exact solution algorithms. Hu et al. [44] also addressed the quadratic shortest path problem, introducing several semidefinite programming relaxations to obtain strong lower bounds. More recently, Granata et al. [38] proposed the notion of *penalized reload cost* in network design, generalizing traditional reload costs by accounting for hidden charges arising from remaining network components after establishing a path, walk, tour, or flow. Their study investigates the computational complexity of these problems and establishes several inapproximability results.

4.4.2 Ride hailing with priorities

The assignment mechanism in ride-hailing platforms can be modeled as a bipartite matching problem, where drivers and ride requests form the two vertex sets. Feasible pairings are represented as edges weighted by the distance or cost of pick-up. The classical approach to minimize total cost is to compute a minimum weight maximum matching, often using the Hungarian algorithm [51].

To incorporate prioritization, the vertex set is partitioned into multiple priority levels, and the assignment seeks to maximize the number of matched agents in higher priority classes before considering the lower ones. This approach aligns with the concept of *priority matchings*, initially studied in kidney exchange problems [67], and later adapted to settings such as school choice [1] and service systems [23].

In ride-hailing systems, priority can be granted to passengers based on loyalty or premium service options, or to drivers who are more reliable or responsive. Studies such as [69] examine the effects of assigning priority to part-time drivers, highlighting the impact on supply stability and matching efficiency. Similarly, [50] analyze the strategic trade-offs in dispatch policies that offer prioritized matching in exchange for reduced per-ride compensation.

Another stream of research explores dynamic and real-time assignment mechanisms. The *Nearest Neighbor* (NN) heuristic, which matches each request with the closest driver, is widely used but criticized for its myopic nature [19]. Batching approaches, which aggregate requests over short intervals before solving the matching problem, provide improved outcomes in terms of reduced average pick-up distances and increased completion rates [87]. These approaches also accommodate threshold constraints, such as the maximum acceptable pick-up distances, to enhance service quality and reduce abandonment [86].

Priority-aware batching algorithms have also been proposed. For instance, two-stage procedures first match priority agents before considering others, aiming to balance priority satisfaction against total cost [62]. These approaches enable platforms to offer differentiated services while maintaining operational efficiency.

4.4.3 Applications of Petri nets in traffic flow and signal control

Traffic congestion is a critical challenge in urban mobility, requiring models that integrate continuous vehicle flows with discrete events such as signal changes and incidents. Petri nets (PN) have emerged as a robust formalism for modeling such systems, as they naturally capture causality, concurrency, conflict, and synchronization. Their bipartite, graphical structure provides both intuitive visualization and rigorous mathematical analysis.

A Petri net is a bipartite directed graph consisting of four elements: places (circles, representing system conditions or resources), transitions (rectangles, representing events), arcs (directed edges linking places and transitions), and tokens (dots residing in places that represent the current state). The distribution of tokens, known as the marking, defines the system's state. System dynamics evolve through the firing of transitions, which consume tokens from input places and produce tokens in output places [15].

Traffic models are typically classified into three levels of detail [20]. *Microscopic models* represent each individual vehicle with high granularity, but are computationally expensive. *Macroscopic models* treat flow as an aggregate stream using variables such as flow, density, and speed, enabling large-scale and real-time applications. *Mesoscopic models* occupy an intermediate position, capturing driver heterogeneity probabilistically.

At the macroscopic level, Continuous Timed Petri Nets (CTPNs) have been used to represent freeway dynamics. Ferreira and Neves [21] proposed a modular CTPN framework where road segments, ramps, and terminals are modeled as reusable subcomponents. Vehicle propagation depends simultaneously on upstream supply and downstream capacity, thereby capturing congestion propagation. To enhance realism, their model introduced parameters for critical density and speed, calibrated through microscopic simulation with Aimsun, achieving a balance between accuracy and computational efficiency.

For intersections, Deterministic and Stochastic Petri Nets (DSPNs) have been employed to represent both stochastic vehicle arrivals and deterministic signal timings. Wang, List, and DiCesare [79] applied DSPNs to model pre-timed and actuated controllers, benchmarking results against Webster's formula [81], Newell's theory [61], and the Highway Capacity Manual (TRB [8]). Their work demonstrated that DSPNs produce reliable delay estimates, particularly in near-saturated conditions.

To address more complex phasing, Timed Coloured Petri Nets (CP-nets) have been developed. Huang and Chung [45] showed that CP-nets could model multi-phase signals compactly while supporting formal safety verification. Using place invariants, they proved that conflicting green phases could not occur in any reachable state. Tristono et al. [74] extended this approach to intersections disrupted by railway crossings, showing that adaptive controllers modeled in CP-nets could clear queues more efficiently than fixed schedules.

Validation in the literature follows two complementary strategies: formal analysis, where techniques such as Occurrence Graphs and Place Invariants prove properties like liveness and safety [45, 53], and simulation benchmarking, where PN outputs are compared with analytical formulas or commercial simulators [21, 74, 79]. Together, these studies confirm that PN models capture both the logic and dynamics of traffic systems with reduced computational cost relative to microscopic simulations.

Petri nets provide a versatile modeling framework across traffic domains. Their modularity and scalability support large networks, their graphical structure improves clarity, and their variants — timed, stochastic, colored, continuous — offer flexibility for different traffic phenomena. The surveyed works collectively highlight Petri nets as an effective tool for intelligent traffic management, balancing formal rigor with practical applicability. For an historic review, see [60].

4.4.4 Circular-arc graph models for traffic-light scheduling

A circular-arc graph is defined as the intersection graph of a family of arcs on a circle, where each vertex corresponds to an arc and two vertices are adjacent if their arcs overlap [75]. These graphs generalize interval graphs and have been widely studied due to their algorithmic tractability and scheduling applications [32]. Characterization problem for the circular-arc graphs first appeared in [40]. In , characterizations and recognition algorithms for the subclasses of circular-arc graphs are given, with emphasis on the linear time algorithms. In traffic control, the idea is adapted by representing traffic movements (straight, left turn, right turn) as vertices in a compatibility graph, where two movements are connected if they can occur simultaneously without conflict [71]. Embedding these movements as arcs around a cycle — where the circumference represents the total signal cycle — yields a circular-arc representation. The corresponding intersection graph captures feasible groups of movements that can be assigned green simultaneously, with maximal cliques representing compatible signal phases.

The modeling framework typically follows four steps: (i) construct the compatibility graph of movements; (ii) map feasible green intervals as arcs around a circle; (iii) derive the intersection (circular-arc) graph, in which cliques correspond to sets of non-conflicting movements; and (iv) solve a linear programming model to allocate green times with the objective of minimizing total waiting or delay. Tanveer [72] applied the approach to a four-leg intersection, showing that embedding streams as arcs and optimizing over cliques significantly reduced average waiting times. Hosseini et al. [43] presented a detailed case study with explicit circular-arc constructions and optimization.

An alternative but related line of work focuses on clique-based analysis directly on the compatibility graph, bypassing explicit arc construction. Here, maximal cliques are interpreted as signal groups, and their sequencing within a fixed cycle is optimized, sometimes aided by clique matrices to systematize feasible group selection. While this approach avoids the circular embedding, it is conceptually similar and relies on the same combinatorial principle of using cliques to identify compatible traffic movements.

From a broader graph-theoretic perspective, circular-arc graphs are a well-characterized class with efficient recognition algorithms and structural results [32, 75]. Applications often include traffic control as a natural domain for circular-arc modeling [43]. While the complement, the co-circular-arc graph, could encode exclusivity constraints, most published implementations of traffic-signal control rely on the intersection model because of its natural mapping to phase compatibility.

More recently, traffic signal research has shifted toward adaptive, data-driven methods such as reinforcement learning and graph neural networks, which prioritize scalability and real-time adaptability [82]. These systems generally move away from explicit circular-arc modeling but often require interpretable initialization plans, for which graph-theoretic methods can still provide value. Thus, circular-arc graph approaches remain relevant for fixed-cycle intersections and as a rigorous, interpretable foundation that complements modern adaptive control.

In summary, circular-arc graphs provide an elegant mathematical framework for traffic-light scheduling, embedding the cyclical nature of signals and the compatibility of movements into a tractable optimization problem. Studies by Tanveer [72] and Hosseini and Orooji [43] demonstrate that such models are effective for minimizing waiting times and structuring phases in fixed-cycle systems, while adaptive approaches dominate large-scale real-time control. Together, these perspectives highlight the complementary roles of classical graph-theoretic methods and modern AI-driven systems in advancing traffic signal research.

Most of the existing studies on circular-arc graphs and related graph-theoretic approaches to traffic-light scheduling focus on specific intersection case studies. While these works demonstrate the feasibility and clarity of the method, they remain limited in scope. To advance the field, there is a need for more general and scalable modeling frameworks that go beyond individual examples and can capture the complexity of urban traffic systems at large.

4.5 Conclusions

This chapter has highlighted major planning and optimization problems in public transportation, including vehicle scheduling, crew scheduling, maintenance planning, and also ride hailing with priorities. Each of these areas poses complex computational challenges that require balancing efficiency, feasibility, and service quality. Traditional sequential and integrated methods have provided valuable progress, yet growing system demands call for innovative approaches. Graph-theoretic methods, such as reload cost in edge-colored graphs, Petri nets for traffic signal modeling, and circular-arc graphs for compatibility analysis, enrich the toolkit available for both researchers and practitioners. Together, these approaches bridge theory and practice, offering new perspectives for designing resilient, efficient, and adaptive public transportation and traffic systems.

Bibliography IV

[1] Atila Abdulkadiroğlu and Tayfun Sönmez. "School choice: A mechanism design approach". In: *American Economic Review* 93.3 (2003), pp. 729–747. doi: 10.1257/000282803322157061.

[2] Edoardo Amaldi, Giulia Galbiati, and Francesco Maoli. "On minimum reload cost paths, tours, and flows". In: *Networks* 57.3 (2011), pp. 254–260. doi: 10.1002/net.20423.

[3] Bastian Amberg, Boris Amberg, and Natalia Kliewer. "Robust efficiency in urban public transportation: Minimizing delay propagation in cost-efficient bus and driver schedules". In: *Transportation Science* 53.1 (2019), pp. 89–112. doi: 10.1287/trsc.2017.0757.

[4] Alejandro Andrade-Michel, Yasmin A. Ríos-Solís, and Vincent Boyer. "Vehicle and reliable driver scheduling for public bus transportation systems". In: *Transportation Research Part B: Methodological* 145 (2021), pp. 290–301. doi: 10.1016/j.trb.2021.01.011.

[5] Julien Baste, Didem Gözüpek, Mordechai Shalom, and Dimitris M. Thilikos. "Minimum reload cost graph factors". In: *Theory of Computing Systems* 65.5 (2021), pp. 815–838. doi: 10.1007/978-3-030-10801-4_7.

[6] Peter Belobaba, Amedeo Odoni, and Cynthia Barnhart. *The Global Airline Industry*. John Wiley & Sons, 2009. doi: ccc.

[7] Alan A. Bertossi, Paolo Carraresi, and Giorgio Gallo. "On some matching problems arising in vehicle scheduling models". In: *Networks* 17.1 (1987), pp. 271–281. doi: 10.1002/net.3230170303.

[8] Transportation Research Board. *Highway Capacity Manual*. Washington, DC: TRB, National Research Council, 2010.

[9] Lawerence Bodin, Bruce Golden, Arjang Assad, and Mark Ball. "Routing and scheduling of vehicles and crews: The state of the art". In: *Computers and Operations Research* 10.1 (1983), pp. 63–212. doi: 10.1016/0305-0548(83)90030-8.

[10] Ralf Borndörfer, Markus Reuther, Thomas Schlechte, Kerstin Waas, and Steffen Weider. "Integrated optimization of rolling stock rotations for intercity railways". In: *Transportation Science* 50.3 (2015), pp. 863–877. doi: 10.1287/trsc.2015.0633.

[11] Khaled Boubaker, Guy Desaulniers, and Issmail Elhallaoui. "Bidline scheduling with equity by heuristic dynamic constraint aggregation". In: *Transportation Research Part B: Methodological* 44.1 (2010), pp. 50–61. doi: 10.1016/j.trb.2009.06.003.

[12] Vincent Boyer, Omar Ibarra-Rojas, and Yasmin Ríos-Solís. "Vehicle and crew scheduling for flexible bus transportation systems". In: *Transportation Research Part B: Methodological* 112 (2018), pp. 216–229. doi: 10.1016/j.trb.2018.04.008.

[13] Yasemin Büyükçolak, Didem Gözüpek, and Sibel Özkan. "Minimum reload cost cycle cover in complete graphs". In: *Networks* 74.3 (2019), pp. 274–286. doi: 10.1002/net.21884.

[14] Narjes Davari, Bruno Veloso, Gustavo de Assis Costa, Pedro Mota Pereira, Rita P. Ribeiro, and João Gama. "A survey on data-driven predictive maintenance for the railway industry". In: *Sensors* 21.17 (2021), paper no. 5739. doi: 10.3390/s21175739.

[15] Rene David and Hassane Alla. *Discrete, Continuous, and Hybrid Petri Nets*. Springer, 2010. doi: 10.1007/978-3-642-10669-9.

[16] Balázs Dávid and Miklós Krész. "Multi-depot bus schedule assignment with parking and maintenance constraints for intercity transportation over a planning period". In: *Transportation Letters* 12.1 (2020), pp. 66–75. doi: 10.1080/19427867.2018.1512216.

[17] Guy Desaulniers and Mark Hickman. "Public transit". In: *Handbooks in Operations Research and Management Science*. Vol. 14. 2007, pp. 69–127. doi: 10.1016/S0927-0507(06)14002-5.

[18] Andreas T. Ernst, Houyuan Jiang, Mohan Krishnamoorthy, Bowie Owens, and D. Sier. "An annotated bibliography of personnel scheduling and rostering". In: *Annals of Operations Research* 127.1 (2004), pp. 21–144. doi: 10.1023/B:ANOR.0000019087.46656.e2.

[19] Yiding Feng, Rad Niazadeh, and Amin Saberi. "Two-stage stochastic matching and pricing with applications to ride hailing". In: *Operations Research* 72.4 (2023), pp. 1574–1594. doi: 10.1287/opre.2022.2398.

[20] Antonella Ferrara, Simona Sacone, and Silvia Siri. *Freeway Traffic Modelling and Control*. Advances in Industrial Control. Springer, 2018. doi: 10.1007/978-3-319-75961-6.

[21] Cláudia Ferreira and Luís Canhoto Neves. "Macro modelling of traffic flow using continuous timed Petri nets". In: *Transportation Planning and Technology* 44.5 (2021), pp. 503–523. doi: 10.1080/03081060.2021.1927304.

[22] Anja Fischer and Christoph Helmberg. "The symmetric quadratic traveling salesman problem". In: *Mathematical Programming* 142.1 (2013), pp. 205–254. doi: 10.1007/s10107-012-0568-1.

[23] Andrew Frazelle and Elena Katok. "Paid priority in service systems: Theory and experiments". In: *Manufacturing & Service Operations Management* 26.2 (2024), pp. 775–795. doi: 10.1287/msom.2021.0387.

[24] Giulia Galbiati. "The complexity of a minimum reload cost diameter problem". In: *Discrete Applied Mathematics* 156.18 (2008), pp. 3494–3497. doi: 10.1016/j.dam.2008.02.013.

[25] Giulia Galbiati, Stefano Gualandi, and Francesco Maffioli. "On minimum changeover cost arborescences". In: *Proceedings of the 10th International Symposium on Experimental Algorithms*. Vol. 6630. 2011, pp. 112–123. doi: 10.1007/978-3-642-20662-7_10.

[26] Giulia Galbiati, Stefano Gualandi, and Francesco Maffioli. "On minimum reload cost cycle cover". In: *Discrete Applied Mathematics* 164 (2014), pp. 112–120. doi: 10.1016/j.dam.2011.12.006.

[27] Michel Gamache, François Soumis, Gérald Marquis, and Jacques Desrosiers. "A column generation approach for large-scale aircrew rostering problems". In: *Operations research* 47.2 (1999), pp. 247–263. doi: 10.1287/opre.47.2.247.

[28] Ioannis Gamvros, Luis Gouveia, and S. Raghavan. "Reload cost trees and network design". In: *Networks* 59.4 (2012), pp. 365–379. doi: 10.1002/net.20443.

[29] Giovanni Luca Giacco, Donato Carillo, Andrea D'Ariano, Dario Pacciarelli, and Ángel G. Marín. "Short-term rail rolling stock rostering and maintenance scheduling". In: *Transportation Research Procedia* 3 (2014), pp. 651–659. doi: 10.1016/j.trpro.2014.10.044.

[30] Giovanni Luca Giacco, Andrea D'Ariano, and Dario Pacciarelli. "Rolling stock rostering optimization under maintenance constraints". In: *Journal of Intelligent Transportation Systems* 18.1 (2014), pp. 95–105. doi: 10.1080/15472450.2013.801712.

[31] Konstantinos Gkiotsalitis, Christina A. Iliopoulou, and Konstantinos L. Kepaptsoglou. "An exact approach for the multi-depot electric bus scheduling problem with time windows". In: *European Journal of Operational Research* 306.1 (2023), pp. 189–206. doi: 10.1016/j.ejor.2022.07.017.

[32] Martin Charles Golumbic. *Algorithmic Graph Theory and Perfect Graphs*. 2nd. Elsevier, 2004.

[33] Laurent Gourvès, Adria Lyra, Carlos Martinhon, and Jerome Monnot. "The minimum reload s-t path, trail and walk problems". In: *Discrete Applied Mathematics* 158.13 (2010), pp. 1404–1417. doi: 10.1016/j.dam.2010.03.009.

[34] Didem Gözüpek, Sibel Özkan, Christophe Paul, Ignasi Sau, and Mordechai Shalom. "Parameterized complexity of the MINCCA problem on graphs of bounded decomposability". In: *Proceddings of the 42nd International Workshop on Graph-Theoretic Concepts in Computer Science*. Vol. 9941. 2016, pp. 195–206. doi: 10.1007/978-3-662-53536-3_17.

[35] Didem Gözüpek, Hadas Shachnai, Mordecahi Shalom, and Shmuel Zaks. "Constructing minimum changeover cost arborescences in bounded treewidth graphs". In: *Theoretical Computer Science* 621 (2016), pp. 22–36. doi: 10.1016/j.tcs.2016.01.022.

[36] Didem Gözüpek and Mordechai Shalom. "Complexity of edge coloring with minimum reload/changeover costs". In: *Networks* 73.3 (2019), pp. 344–357. doi: 10.1002/net.21867.

[37] Didem Gözüpek, Mordechai Shalom, Ariella Voloshin, and Shmuel Zaks. "On the complexity of constructing minimum changeover cost arborescences". In: *Theoretical Computer Science* 540 (2014), pp. 40–52. doi: 10.1016/j.tcs.2014.03.023.

[38] Donatella Granata. "On penalized reload cost path, walk, tour and maximum flow: hardness and approximation". In: *Optimization Letters* 19.1 (2025), pp. 123–138. doi: 10.1007/s11590-024-02108-x.

[39] Ahmed Hadjar, Odile Marcotte, and François Soumis. "A Branch-and-Cut Algorithm for the Multiple Depot Vehicle Scheduling Problem". In: *Operations Research* 54.1 (2006), pp. 130–149. doi: 10.1287/opre.1050.0240.

[40] Hugo Hadwiger, Hans Debrunner, and Victor Klee. *Combinatorial Geometry in the Plane*. New York: Holt, Rinehart and Winston, 1964.

[41] Ali Haghani and Yousef Shafahi. "Bus maintenance systems and maintenance scheduling: model formulations and solutions". In: *Transportation Research Part A: Policy and Practice* 36.5 (2002), pp. 453–482. doi: 10.1016/S0965-8564(01)00014-3.

[42] Yanbo He, Wenyi Zhang, Tao Liu, Jihui Ma, and Huijun Sun. "Day-to-Day Integrated Optimization of Bus Transit Maintenance and Vehicle Scheduling". In: *IEEE Transactions on Intelligent Transportation Systems* 26.3 (2025), pp. 3814–3830. doi: 10.1109/TITS.2024.3519345.

[43] Seyed Mohsen Hosseini and H Orooji. "Phasing of traffic light at a road junction". In: *Applied Mathematical Science* 3.30 (2009), pp. 1487–1492. url: x.

[44] Hao Hu and Renata Sotirov. "On solving the quadratic shortest path problem". In: *INFORMS Journal on Computing* 32.2 (2020), pp. 219–233. doi: 10.1287/ijoc.2018.0861.

[45] Yi-Sheng Huang and Ta-Hsiang Chung. "Modelling and analysis of traffic light control systems using timed coloured Petri nets". In: *Petri Nets Applications*. InTech, 2010. Chap. 27, pp. 567–588. doi: 10.5772/7497.

[46] Dennis Huisman, Richard Freling, and Albert P.M. Wagelmans. "Multiple-depot integrated vehicle and crew scheduling". In: *Transportation Science* 39.4 (2005), pp. 491–502. DOI: 10.1287/trsc.1040.0104.

[47] O.J. Ibarra-Rojas, F. Delgado, R. Giesen, and J.C. Muñoz. "Planning, operation, and control of bus transport systems: A literature review". In: *Transportation Research Part B: Methodological* 77 (2015), pp. 38–75. DOI: 10.1016/j.trb.2015.03.002.

[48] Liujien Kang, Qiang Meng, and Chuanbei Zhou. "Bus driver scheduling enhancement: A derandomizing approach for uncertain bus trip times". In: *Transportmetrica B: Transport Dynamics* 8.1 (2020), pp. 200–218. DOI: 10.1080/21680566.2019.1695153.

[49] Natalia Kliewer, Bastian Amberg, and Boris Amberg. "Multiple depot vehicle and crew scheduling with time windows for scheduled trips". In: *Public Transport* 3.3 (2012), pp. 213–244. DOI: 10.1007/s12469-011-0049-6.

[50] Varun Krishnan, Ramon Iglesias, Sebastien Martin, Su Wang, Varun Pattabhiraman, and Garrett Van Ryzin. "Solving the ride-sharing productivity paradox: Priority dispatch and optimal priority sets". In: *INFORMS Journal on Applied Analytics* 52.5 (2022), pp. 433–445. DOI: 10.1287/inte.2022.1134.

[51] H. W. Kuhn. "The Hungarian method for the assignment problem". In: *Naval Research Logistics Quarterly* 2.1-2 (1955), pp. 83–97. DOI: 10.1002/nav.3800020109.

[52] Yung-Cheng Lai, Dow-Chung Fan, and Kwei-Long Huang. "Optimizing rolling stock assignment and maintenance plan for passenger railway operations". In: *Computers & Industrial Engineering* 85 (2015), pp. 284–295. DOI: 10.1016/j.cie.2015.03.016.

[53] George F. List and Mecit Cetin. "Modeling traffic signal control using Petri nets". In: *IEEE Transactions on Intelligent Transportation Systems* 5.3 (2004), pp. 177–187. DOI: 10.1109/TITS.2004.833763.

[54] Andreas Löbel. "Vehicle scheduling in public transit and Lagrangean pricing". In: *Management Science* 44.12-part-1 (1998), pp. 1637–1649. DOI: 10.1287/mnsc.44.12.1637.

[55] Pedro Cesar Lopes Gerum, Ayca Altay, and Melike Baykal-Gürsoy. "Data-driven predictive maintenance scheduling policies for railways". In: *Transportation Research Part C: Emerging Technologies* 107 (2019), pp. 137–154. DOI: 10.1016/j.trc.2019.07.020.

[56] Helena Lourenço, Jose Paixão, and Rita Portugal. "Multiobjective metaheuristics for the bus driver scheduling problem". In: *Transportation Science* 35.3 (2001), pp. 331–343. DOI: 10.1287/trsc.35.3.331.10147.

[57] Jihui Ma, Cuiying Song, Avishai Ceder, Tao Liu, and Wei Guan. "Fairness in optimizing bus-crew scheduling process". In: *PLOS One* 12.11 (2017), e0187623. DOI: 10.1371/journal.pone.0187623.

[58] Lucas Mertens, Lena-Antonia Wolbeck, David Rössler, Lin Xie, and Natalia Kliewer. *An overview of optimization approaches for scheduling and rostering resources in public transportation*. Tech. rep. URL: https://arxiv.org/abs/2310.13425 (visited on 09/26/2025).

[59] Marta Mesquita and Ana Paias. "Set partitioning/covering-based approaches for the integrated vehicle and crew scheduling problem". In: *Computers & Operations Research*. Part Special Issue: Algorithms and Computational Methods in Feasibility and Infeasibility 35.5 (2008), pp. 1562–1575. DOI: 10.1016/j.cor.2006.09.001.

[60] Tadao Murata. “Petri nets: Properties, analysis and applications”. In: *Proceedings of the IEEE* 77.4 (1989), pp. 541–580. doi: 10.1109/5.24143.

[61] G. F. Newell. “Approximation methods for queues with application to the fixed-cycle traffic light”. In: *SIAM Review* 7.2 (1965), pp. 223–240. doi: 10.1137/1007038.

[62] Yasunori Okumura. “Priority Matchings Revisited”. In: *Games and Economic Behavior* 88 (2014), pp. 242–249. doi: 10.1016/j.geb.2014.10.007.

[63] Hande Öztop, Ugur Eliiyi, Deniz Eliiyi, and Levent Kandiller. “A bus crew scheduling problem with eligibility constraints and time limitations”. In: *Transportation Research Procedia* 22 (2017), pp. 222–231. doi: 10.1016/j.trpro.2017.03.029.

[64] David Pardo Peña, David Alvarez Martinez, and John Escobar. “A GRASP algorithm for the bus crew scheduling problem”. In: *International Journal of Industrial Engineering Computations* (2023). doi: 10.5267/j.ijiec.2024.1.003.

[65] Shyam S.G. Perumal, Twan Dollevoet, Dennis Huisman, Richard M. Lusby, Jesper Larsen, and Morten Riis. “Solution approaches for integrated vehicle and crew scheduling with electric buses”. In: *Computers & Operations Research* 132 (2021), paper no. 105268. doi: 10.1016/j.cor.2021.105268.

[66] Borzou Rostami, André Chassein, Michael Hopf, Davide Frey, Christoph Buchheim, Federico Malucelli, and Marc Goerigk. “The quadratic shortest path problem: Complexity, approximability, and solution methods”. In: *European Journal of Operational Research* 268.2 (2018), pp. 473–485. doi: 10.1016/j.ejor.2018.01.054.

[67] Alvin Roth, Tayfun S "onmez, and Utku Ünver. “Kidney exchange”. In: *The Quarterly Journal of Economics* 119.2 (2004), pp. 457–488. doi: 10.1162/0033553041382157.

[68] Yindong Shen, Jia Xu, and Jingpeng Li. “A probabilistic model for vehicle scheduling based on stochastic trip times”. In: *Transportation Research Part B: Methodological* 85 (2016), pp. 19–31. doi: 10.1016/j.trb.2015.12.016.

[69] Junxin Shi, Xiangyong Li, Yash Pal Aneja, and Xiaonan Li. “Ride-matching for the ride-hailing platform with heterogeneous drivers”. In: *Transport Policy* 136 (2023), pp. 169–192. doi: 10.1016/j.tranpol.2023.04.001.

[70] Ingmar Steinzen, Vitali Gintner, Leena Suhl, and Natalia Kliewer. “A time-space network approach for the integrated vehicle- and crew-scheduling problem with multiple depots”. In: *Transportation Science* 44.3 (2010), pp. 367–382. doi: 10.1287/trsc.1090.0304.

[71] Karl E. Stoffers. “Scheduling of traffic lights – A new approach”. In: *Transportation Research* 2.3 (1967), pp. 199–234. doi: 10.1016/0041-1647(68)90016-6.

[72] Shakera Tanveer. “Application of graph theory in representing and modelling traffic control problems”. In: *International Journal of Mathematics and Computer Applications Research* 6.3 (2016), pp. 29–34. url: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2838642 (visited on 09/29/2025).

[73] Nour ElHouda Tellache, Frédéric Meunier, and Axel Parmentier. “Linear lexicographic optimization and preferential bidding system”. In: *Transportation Science* 58.3 (2024), pp. 597–613. doi: 10.1287/trsc.2022.0372.

[74] Tomi Tristono, Setiyo Daru Cahyono, Sutomo, and Pradityo Utomo. "Model Petri net of adaptive traffic lights and its collaboration with a special event". In: *MATEC Web of Conferences*. Vol. 147. 2018, paper no. 02005. DOI: 10.1051/matecconf/201814702005.

[75] Alan Tucker. "Structure theorems for some circular-arc graphs". In: *Discrete Mathematics* 7.1 (1974), pp. 167–195. DOI: 10.1016/S0012-365X(74)80027-0.

[76] B. T. C. Van Rossum, T. Dollevoet, and D. Huisman. "Railway crew planning with fairness over time". In: *European Journal of Operational Research* 318.1 (2024), pp. 55–70. DOI: 10.1016/j.ejor.2024.04.029.

[77] Bruno Veloso, Rita P. Ribeiro, João Gama, and Pedro Mota Pereira. "The MetroPT dataset for predictive maintenance". In: *Scientific Data* 9.1 (2022), paper no. 764. DOI: 10.1038/s41597-022-01877-3.

[78] Victor Andrés Vera Valdes. *Integrating Crew Scheduling and Rostering Problems*. PhD thesis. 2010. DOI: 10.6092/unibo/amsdottorato/2705.

[79] Huaitien Wang, George F. List, and Frank DiCesare. "Modeling and evaluation of traffic signal control using timed Petri nets". In: *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*. Vol. 2. 1993, pp. 180–185. DOI: 10.1109/ICSMC.1993.384868.

[80] Jing Wang, Heqi Wang, Ande Chang, and Chen Song. "Collaborative optimization of vehicle and crew scheduling for a mixed fleet with electric and conventional buses". In: *Sustainability* 14.6 (2022), p. 3627. DOI: 10.3390/su14063627.

[81] F. V. Webster. *Traffic Signal Settings. Road Research Technical Paper No. 39*. Tech. rep. 1958. URL: https://www.sinaldetransito.com.br/artigos/traffic_signals_webster.pdf (visited on 09/26/2025).

[82] Hua Wei, Guanjie Zheng, Huaxiu Yao, and Zhenhui Li. "IntelliLight: A reinforcement learning approach for intelligent traffic light control". In: *Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*. 2019, pp. 2496–2505. DOI: 10.1145/3219819.3220096.

[83] Hans-Christoph Wirth and Jan Steffen. "Reload cost problems: Minimum diameter spanning tree". In: *Discrete Applied Mathematics* 113.1 (2001), pp. 73–85. DOI: 10.1016/S0166-218X(00)00392-9.

[84] Weitiao Wu and Yangyi Zhu. "Safety-oriented multi-depot integrated vehicle and crew scheduling problem". In: *Transportation Research Part E: Logistics and Transportation Review* 203 (2025), paper no. 104348. DOI: 10.1016/j.tre.2025.104348.

[85] Lin Xie, Marius Merschformann, Natalia Kliewer, and Leena Suhl. "Metaheuristics approach for solving personalized crew rostering problem in public bus transit". In: *Journal of Heuristics* 23.5 (2017), pp. 321–347. DOI: 10.1007/s10732-017-9348-7.

[86] Hai Yang, Xiaoran Qin, Jintao Ke, and Jieping Ye. "Optimizing matching time interval and matching radius in on-demand ride-sourcing markets". In: *Transportation Research Part B: Methodological* 131 (2020), pp. 84–105. DOI: 10.1016/j.trb.2019.11.005.

[87] Ruiling Yang and Minghui Lai. "Multi-Level Analyses on the Nearest-First Matching Policy of On-Demand Chauffeured Ride-Hailing Service". In: *International Journal of Sustainable Transportation* 15.10 (2021), pp. 749–767. DOI: 10.1080/15568318.2020.1795760.

Chapter V

Disaster Management

Combinatorial optimization is critical for improving disaster management operations, spanning pre-disaster planning, shelter allocation, humanitarian logistics, and evacuation strategies. This chapter reviews the literature across these domains, highlighting methodological advances, practical applications, and research gaps. In pre-disaster planning, transportation infrastructure is assessed using risk, vulnerability, resilience, and network-theoretic metrics, with graph-theoretic optimization largely unexplored. Shelter allocation and layout design are analyzed through deterministic and stochastic models, incorporating fairness and dynamic demand, while joint optimization with evacuation planning is emerging. Humanitarian logistics networks are examined via single- and multi-level facility location models, emphasizing cost, service efficiency, and robustness under uncertainty. Graph-theoretic approaches show promise for designing resilient multi-level supply chains and post-disaster distribution. Evacuation planning includes pedestrian- and transit-based models, multi-objective optimization, and equity-focused algorithms. Across all areas, integrating theoretical graph models, uncertainty, and multi-modal considerations presents significant opportunities for future research.

Keywords: Disaster management, shelter planning, infrastructure design in pre-disaster management, humanitarian logistics network, evacuation planning, graph theory, combinatorial optimization.

V

Combinatorial Optimization in Disaster Management

Michael Ayomoh, University of Pretoria, South Africa
Didem Gözüpek, Gebze Technical University, Türkiye
Sibel Özkan, Gebze Technical University, Türkiye
Joanna Raczek, Gdańsk University of Technology, Poland
Fatih Yetgin, Gebze Technical University, Türkiye
Paweł Żyliński, University of Gdańsk, Poland

Disasters, whether natural, technological, or man-made, pose severe challenges to human life, critical infrastructure, and societal functions, particularly in rapidly urbanizing areas, highlighting the urgent need to design and maintain disaster-resilient cities. The intensifying impacts of climate change not only increase the frequency and severity of disasters but also exacerbate their effects on urban populations and infrastructure.

Effective disaster management requires timely, coordinated, and resource-efficient interventions across multiple operational domains, from preparedness and mitigation to response and recovery. In recent decades, combinatorial optimization has emerged as a powerful tool for addressing these challenges, providing systematic methods to allocate scarce resources, design resilient networks, and plan complex evacuation and relief operations [40].

The application of combinatorial optimization in disaster management is particularly compelling because disasters are characterized by high uncertainty, complex interdependencies, and time-critical decision requirements. Unlike traditional optimization problems, disaster management scenarios must account for unpredictable demand surges, disrupted infrastructure, and dynamic behavioral responses from affected populations. This makes the development of robust, flexible, and equitable optimization models both scientifically challenging and practically vital.

This chapter reviews the state-of-the-art applications of combinatorial optimization in disaster management along four major dimensions: In Section 5.1 we review infrastructure design in pre-disaster scenarios, in Section 5.2 we focus on shelter allocation and layout design, in Section 5.3 we discuss the related literature on humanitarian logistics networks, and in Section 5.3.3 we review the literature on evacuation planning. Finally, we conclude this chapter in Section 5.4.

5.1 Infrastructure design in pre-disaster scenarios

Transportation infrastructure systems are critical lifelines that must be designed and managed with potential disruptions in mind. They not only enable the daily movement of people and goods, connect businesses, and sustain supply chains, but they also serve as the backbone for emergency access and recovery operations when disasters strike. From a pre-disaster perspective, their performance lies in ensuring continuity of essential services, providing redundancy, and maintaining flexibility to withstand or adapt to unexpected hazards. Lack of preparedness or weak links in these systems can amplify disaster impacts, highlighting the need for a proactive assessment of critical components and investment in resilient infrastructure.

There is no single standard formalization of link criticality in transportation networks. Jenelius et al. [38] adopt a risk-based view, defining criticality as a function of both failure likelihood and consequence severity.

In contrast, De Oliveira et al. [20] treat criticality as probability-neutral. These differing approaches have produced a wide variety of metrics, ranging from basic road capacity measures [68] to advanced connectivity-based indicators [46].

Metrics can be broadly grouped into two categories [37]: those originating in transportation studies and those derived from network theory. Examples from transportation studies include change in weighted total travel cost [5, 21], change in worst-case user exposure [52], traffic flow [75], and traffic density [62, 75]. Metrics inspired by network theory include unweighted betweenness centrality [22, 43], minimum link cut centrality [67], and OD k-connectivity [53, 66]. For a comprehensive review of metrics, see [37].

From a disaster management perspective, Faturechi and Miller-Hooks [27] classify performance measures into risk, vulnerability, reliability, robustness, flexibility (agility and adaptability), survivability, and resilience.

- Risk combines the probability of an event with its consequences in terms of system performance.
- Vulnerability denotes the susceptibility of the system to threats that cause operational degradation.
- Reliability refers to the probability that the system continues operating at a satisfactory level after a disaster.
- Robustness is the ability to withstand or absorb disruptions and remain intact.
- Flexibility captures the system's adaptability through contingency measures.
- Survivability reflects the system's capacity to endure sudden disturbances while meeting demand.
- Resilience is the ability to resist, absorb, adapt, and return to normal functionality after disruptions.

These concepts have been applied widely in disaster-oriented infrastructure research (see, for example, [15, 27, 33, 51, 59]).

Research has also examined other types of infrastructure, such as hospital systems and their responses to disasters [64]. In addition, studies such as [3] on maritime port networks highlight resiliency and reliability from a different network perspective.

Considering this body of literature, it is evident that while graph-theoretic concepts such as betweenness centrality and connectivity have been applied, the use of graph-based metrics alongside graph-theoretic optimization problems remains largely unexplored in the context of link criticality in disaster management. This represents a significant and promising research opportunity.

5.2 Shelter allocation and layout design

The increasing frequency and severity of disasters underscore the urgent need for robust, survivor-focused planning strategies. Among these, the provision of rapid and effective shelter solutions is a cornerstone of disaster management. Most existing approaches in the literature concentrate on determining optimal shelter locations, typically framed as facility location problems.

However, in the aftermath of large-scale disasters, shelter sites often resemble refugee camps, requiring extensive space and careful spatial organization. Despite their practical significance, the layout planning of such large-scale camp sites remains a significantly underexplored area, particularly from an optimization perspective.

In this subsection, we first provide a concise overview of facility location-type problems within the disaster management literature. We then examine related studies on shelter placement, including research that integrates shelter placement with evacuation routes. Finally, we explore the limited but growing body of work on shelter and camp layout planning, and outline promising directions for future research in this critical area.

Humanitarian support facilities can be functionally classified into suppliers, distribution centers, points of distribution, shelters, field hospitals, and blood centers [23]. For instance, the work in [16] focuses on optimally determining the locations of temporary emergency medical services by considering post-disaster transportation infrastructure damage and increased demand together with a case study in New Taipei City. Likewise, the authors in [57] propose a two-stage stochastic model to optimize the location of temporary medical centers for disaster response, minimizing setup and casualty transportation costs while accounting for triage, existing hospitals, and possible infrastructure damage together with a case study in Kartal district of İstanbul. In addition, some studies such as [73] focus on the location allocation of health services networks where the facilities are subject to the risk of disruptions.

An illustrative study on the allocation of blood centers in both pre- and post-disaster scenarios is presented in [65], where the authors propose a strategic framework for optimizing the placement and number of temporary blood facilities. The approach integrates a minimax facility location model with a tabu search heuristic to minimize the maximum distance between temporary centers and hospitals, thereby enhancing response times during emergencies. For a comprehensive review of humanitarian facility location models under uncertainty, we refer the reader to [23].

Beyond medical and blood-related services, the strategic allocation of shelters plays a central role in post-disaster humanitarian logistics, as it directly affects the well-being and safety of displaced populations. London Resilience Forum [50] identifies two types of shelters: Emergency Evacuation Centers (EECs) and Emergency Rest Shelters. An EEC is defined as a facility that provides short-term shelter for individuals unable to secure alternative accommodations. While EECs may offer limited food provisions, they typically do not include sleeping areas. On the other hand, an ERC refers to a facility that offers basic services such as food, washing facilities, and designated sleeping spaces for those without access to other housing options. Given the functional differences between these shelter types, effectively allocating individuals to appropriate facilities requires not only accurate classification, but also careful consideration of spatial, infrastructural, and logistical constraints. This brings us to the broader challenge of identifying suitable shelter locations and evaluating their feasibility.

The Turkish Red Crescent defines 10 criteria to rank potential shelter areas: (1) Transportation of relief items, (2) Procurement of relief items, (3) Healthcare institutions, (4) Topography of the terrain, (5) Type of terrain, (6) Slope of the terrain, (7) Electrical infrastructure, (8) Sanitary system, (9) Flora of the terrain, and (10) Ownership. While the Turkish Red Crescent's criteria provide a solid foundation, the method suffers from two major issues: it overlooks the distance between demand points and shelters, potentially assigning people to faraway locations, and it fails to ensure balanced use of shelter capacities, resulting in some shelters being overcrowded while others remain underused.

To address these shortcomings, Kılcı et al. [44] proposed a mixed integer linear programming (MILP)-based methodology for selecting the location of temporary shelter sites, maximizing the minimum weight of open shelter areas while simultaneously determining shelter locations, population assignments, and utilization control. The model developed by Kılcı et al. introduces fairness-based constraints to regulate usage levels, including parameters that bound both minimum utilization and inter-shelter occupancy differences. The model is applied to real data from the Kartal district of Istanbul, Türkiye, and tested under scenarios from the 2011 Van earthquake, demonstrating its practical applicability in designing equitable and efficient emergency shelter systems. Kılcı et al. propose a deterministic MILP model that incorporates fairness and accessibility in shelter site selection; the model assumes demand is known with certainty, which is a strong simplification that does not reflect the unpredictable nature of real-world disasters.

Addressing this limitation, the study by Kınay et al. [45] extends the earlier deterministic framework by incorporating demand uncertainty through a chance-constrained programming approach. The proposed model ensures, with high probability, that shelter capacity and utilization constraints are satisfied, avoiding both overpreparation and underprotection. Using the central limit theorem, they derive a deterministic approximation of the chance constraints and formulate a tractable MILP. The model is tested using real-world data from the Kartal district and also scaled up to cover the entire Anatolian side of Istanbul, Türkiye. The results show that explicitly modeling uncertainty can significantly alter shelter location decisions and lead to more resilient and realistic disaster preparedness plans.

The paper [48] focuses on hierarchical earthquake shelter planning in urban areas, specifically using Shanghai, China, as a case study. The authors propose a two-stage mathematical programming model that addresses both shelter location and victim allocation, aiming to minimize construction costs and evacuation distances. A key innovation is their consideration of time-varying refuge demand, acknowledging that the number of victims and their needs change throughout the post-earthquake period. The research demonstrates that this dynamic approach to planning can lead to a more cost-effective and efficient disaster response compared to traditional methods that assume constant demand.

The paper [25] discusses the shelter site selection problem for earthquake preparedness, focusing on Istanbul, Türkiye. It highlights the limitations of traditional deterministic and stochastic optimization models, which often fail to adequately account for the uncertainty inherent in disaster scenarios, particularly concerning the fluctuating demand for shelters. The paper proposes a novel robust optimization approach that more accurately predicts shelter demand by integrating seismological data and considers multiple potential earthquake scenarios, emphasizing solutions that are socially acceptable and preferable by maximizing fairness and minimizing walking distances to shelters, even if it means opening more facilities. The findings demonstrate that this robust model consistently provides feasible and effective solutions across a range of possible disaster outcomes, offering a superior method for disaster preparedness planning.

The paper [74] presents a scenario-based, multi-objective optimization model for allocating earthquake emergency shelters. The study focuses on Chaoyang District, Beijing, China, aiming to minimize both total evacuation time and total shelter area while considering factors like shelter capacity and service radius. To address the problem's complexity, the authors modified the particle swarm optimization (PSO) algorithm by incorporating a simulated annealing (SA) algorithm and an outside loop, which improved the algorithm's ability to find optimal solutions and avoid premature convergence. The findings indicate that increasing shelter area can significantly reduce evacuation time, offering practical insights for disaster planning and resource allocation in urban environments.

The work [17] presents a hierarchical location model for planning earthquake shelters. The authors propose classifying shelters into immediate, short- and long-term categories based on changing survivor needs over time following an earthquake. The article outlines a mathematical model to optimize shelter placement by minimizing travel distance and considering financial constraints, using a case study in Beijing, China, to illustrate its practical application. The findings emphasize the importance of budget planning and flexible resource allocation in emergency response.

The paper [71] introduces a multi-criteria model for locating earthquake evacuation shelters, specifically designed for urban planning. Recognizing that traditional models often simplify complex needs, the authors propose a new approach based on seven crucial urban planning principles, such as safety, proximity to residents, and economic viability. The paper details a GIS-supported iterative solution method to implement this model, demonstrating its practical application and effectiveness through a case study in Yangzhou, China.

Until 2011, the emphasis in the literature was to model shelter allocation and evacuation routing as separate problems. In 2011, the seminal paper [10] introduced the Bus Evacuation Problem (BEP), a variant of the Vehicle Routing Problem designed for large-scale evacuations. It involves routing a fleet of identical, capacity-limited buses from yards to pick up evacuees and deliver them to shelters with limited capacity. The aim is to minimize the total evacuation time, from the first bus departure to the last evacuee's arrival, while handling both bus and shelter capacity constraints.

Subsequent studies have refined the joint shelter allocation and evacuation modeling by also integrating demand uncertainty. Li et al. [47] developed a bi-level stochastic program coupling shelter decisions with dynamic user equilibrium for hurricanes in North Carolina. The work [32] uniquely combined car and bus-based evacuations under a system optimal framework, tested in Kaiserslautern and Nice, while the authors in [7] applied a constrained system optimal approach to balance fairness and efficiency in Istanbul earthquake scenarios. The work [29] integrated shelter siting, warnings, and routing for flood evacuations in North Carolina, whereas the work [34] modeled cost-minimizing sink locations under capacity limits in Kaiserslautern. The authors in [63] addressed bushfire bus evacuations in Australia with a multi-objective model prioritizing safety and resource efficiency. Finally, the authors in [8] extended stochastic modeling to capture uncertain demand and infrastructure disruptions in Istanbul earthquakes.

Table 5.1 presents a comparative summary of the papers discussed in this subsection. As noted earlier, most combinatorial optimization studies on shelters in disaster management focus primarily on determining shelter locations and/or assigning survivors to shelters, while largely neglecting the design and layout of shelter sites. Yet, in post-disaster contexts, shelter areas often resemble refugee camps. To address living conditions in such settings, the United Nations Refugee Agency has developed planning standards [69]. To the best of our knowledge, the only study explicitly addressing the optimization of shelter layout design is [41], where the authors employ a general block design structure to account for facility requirements and their interrelations.

5.3 Humanitarian logistics networks

Humanitarian logistics involves the distribution of relief in the aftermath of disasters; therefore, the design and management of humanitarian relief logistics networks are of critical importance. Most studies in this area concentrate on facility location problems, while some integrate these with other logistics challenges such as relief distribution, casualty transportation, and evacuation planning [14].

Most humanitarian relief distribution networks are modeled as single-level structures and can be broadly classified into two categories: minisum facility location problems [36, 49], which aim to minimize the total distance and/or facility-related costs, and set-covering or maximal coverage problems [39], which seek to minimize the number or cost of facilities while ensuring that all demand points are covered.

On the other hand, several humanitarian relief distribution networks, such as those studied in [11, 31, 54], adopt a multi-level structure, falling under the broad category of multi-echelon supply chain networks [13, 55]. In such networks, suppliers of relief items constitute the first level, with entry points into the disaster-affected region typically being airports, seaports, or designated helicopter landing sites. The second level comprises relief distribution centers, functioning as depots, while the third level corresponds to shelters housing displaced populations who require emergency assistance. These settings can also be framed as multi-level facility location problems, which we discuss in greater detail in the next subsection.

5.3.1 Multi-level facility location problem

The multi-level facility location problem, extensively studied in the literature, has been addressed through various discrete optimization methods across several of its versions. Hierarchical facility location models formulate the challenge of placing facilities across multiple tiers within a network, typically aiming to optimize both service efficiency and associated costs. A key aspect of these models is the flow of customers or goods between different levels of the hierarchy. These models are used in various applications, including healthcare, logistics, computer networks, and many others.

Table 5.1: Comparative summary of humanitarian facility location studies.

Citation	**Facility Type**	**Focus / Objective**	**Modeling Approach**	**Uncertainty Consideration**	**Evacuation Consideration**	**Case Study**	**Notable Features**
Chen & Yu (2016) [16]	Field Hospitals	Location of EMS considering infrastructure damage and demand surges	Mathematical optimization	Yes (infrastructure disruption)	No	New Taipei City, Taiwan	Incorporates post-disaster road conditions
Oksuz et al. (2020) [57]	Medical Centers	Minimize setup and casualty transport costs; includes triage, hospitals	Two-stage stochastic programming	Yes (demand, infrastructure)	No	Kartal, Istanbul, Türkiye	Realistic disaster healthcare features
Zarrinpoor et al. (2017) [73]	Health Networks	Health service network allocation under disruption risk	Reliability-based location modeling	Yes (facility disruptions)	No	Not specified	Resilient facility network design
Sharma et al. (2019) [65]	Blood Centers	Minimize max distance between blood centers and hospitals	Minimax location + tabu search	No	No	Not specified	Pre- and post-disaster blood logistics
Kılcı et al. (2015) [44]	Shelters	Fair and efficient shelter site selection and population assignment	MILP	No (deterministic demand)	No	Kartal, Istanbul, Türkiye	Fairness-based constraints for balanced utilization
Kınay et al. (2018) [45]	Shelters	Extend deterministic model with demand uncertainty	Chance-constrained MILP	Yes (demand uncertainty)	No	Anatolian side, Istanbul	Deterministic approximation using CLT
Li et al. (2017) [48]	Hierarchical Shelters	Minimize construction cost and evacuation distance with time-varying demand	Two-stage mathematical programming	Yes (dynamic demand)	No	Shanghai, China	Captures evolving post-earthquake demand
Eriskin et al. (2024) [25]	Shelters	Robust shelter planning under multiple earthquake scenarios	Robust optimization	Yes (multi-scenario uncertainty)	No	Istanbul, Türkiye	Incorporates seismological data and fairness
Zhao et al. (2015) [74]	Shelters	Minimize evacuation time and shelter area	Multi-objective optimization + PSO+SA	Yes (scenario-based)	No	Chaoyang, Beijing, China	Hybrid metaheuristics to enhance solution quality
Chen et al. (2013) [17]	Temporal Shelters	Multi-phase shelter planning (immediate to long-term)	Hierarchical location model	No	No	Beijing, China	Budget-aware, time-sensitive shelter classification
Xu et al. (2016) [71]	Evacuation Shelters	Urban planning-based shelter site selection using 7 criteria	MCDA + GIS-supported method	No	No	Yangzhou, China	Iterative, principle-driven site selection
Bish et al. (2011) [10]	Bus yards, pick-up locations and shelters	Minimize the total evacuation time	Mixed integer programming + heuristic algorithms	No	Yes	New Orleans after Hurricane Katrina	Focus on transit dependent populations, split deliveries allowed
Li et al. (2012) [47]	Shelters	Optimize the shelter locations	Scenario-based bi-level programming	Yes (two-stage stochastic programming)	Yes	North Carolina, USA	Incorporation of user equilibrium-based traffic dynamics
Goerigk et al. (2014) [32]	Shelters and bus depots	Minimize the total evacuation time	Multi-criteria mixed integer programming	No	Yes	Kaiserslautern, Germany and Nice, France	A comprehensive integrated model
Bayram et al. (2016) [7]	Shelters	Minimize the total evacuation time	Nonlinear and second-order conic mixed integer programming	No	Yes	Sioux Falls and Istanbul	Detailed efficiency and fairness metrics

Citation	Facility Type	Focus / Objective	Modeling Approach	Uncertainty Consideration	Evacuation Consideration	Case Study	Notable Features
Gama et al. (2016) [29]	Shelters	Minimize the evacuees' total travel distance the shelters	Multi-period location allocation with mixed integer linear programming	Yes (temporal prediction of the magnitude and evolution of the flood)	Yes	Wake County, North Carolina, USA	Novel dynamic integration of decisions
Hessler et al. (2016) [34]	Emergency shelters	Select a minimum cost subset of nodes as shelters	mixed integer programming and source location algorithms	No	Yes	Kaiserslautern, Germany	Detailed 3-position classi
Shahparvari et al. (2016) [63]	Assembly points and shelters	Maximize the number of evacuated people and minimize the allocated resources	Multi-objective integer linear programming	Yes	Yes	Murrindindi Shire in Victoria, Australia,	Focus on late evacuees
Bayram et al. (2017) [8]	Shelters	Minimize the expected total evacuation time	Stochastic mixed integer non-linear programming	Yes	Yes	İstanbul	Exact Benders decomposition-based algorithm for large scale stochastic problems

Hierarchical facility location models involve the strategic placement of facilities at multiple levels — such as local, regional, and central — to serve customer demands, with the objective of minimizing operational and transportation costs while maintaining adequate service quality across all tiers. This class of problems is referred to in the literature by various terms, including multi-echelon, multi-stage, multi-level, hierarchical, and multi-layer facility location problems, all of which denote what we collectively define as hierarchical facility location models.

Several types of hierarchical facility location models have been proposed in the literature. Flow-based models consider the movement of demand through the hierarchical structure, with each facility level responsible for processing specific portions of the total demand [26]. Assignment-based models allocate demand to facilities at each level, often following predefined paths [9, 30]. Median models aim to minimize the weighted distance between customers and facilities [9, 70], while covering models focus on ensuring that all demand points are located within a specified distance or time threshold from a facility [18, 24, 30].

Two comprehensive review papers summarize the development of hierarchical facility location models (HFLMs): one published in 2007, covering literature up to 2004 [61], and another published in 2018, reviewing advancements up to that year [58]. The classification scheme and terminology introduced in [61] situate multi-level facility location problems within a broader framework, based on key criteria, such as: flow patterns, service availability, and spatial configuration, which correlate with the variation of models stated in the previous paragraph. Building on the extensive body of existing research, we focus in the following sections on recent advances reported in the contemporary literature.

The work [70] proposes a novel median model for the allocation of oncological treatment units within Brazil's Unified Health System. Specifically, the study examines a two-level hierarchical uncapacitated median model, incorporating additional constraints on maximum distance and vertex eligibility. The authors further implement and evaluate the model using CPLEX on real-world data from Rio de Janeiro.

Similarly, the work [72] investigates dynamic service facility location and allocation under time-varying demand. The study emphasizes strategic deployment of service centers and dynamic customer allocation and planning. To determine optimal station locations, the authors employ two hierarchical facility location models and use a geographic information system (GIS) to assess the spatial distribution of potential trip demand. Additionally, they formulate a binary integer programming model and propose a genetic algorithm.

The paper [30] addresses an integrated problem of hierarchical facility location and network design, encompassing multiple decisions regarding the opening of facilities and network links across various levels. The proposed multi-period model incorporates budgetary constraints and optimizes transportation network links within each time period. Its objective is to determine the optimal upgrade levels for urban centers and network links while adhering to a predefined budget. The model is formulated as a mixed-integer linear programming problem.

The work [4] introduces a three-level hierarchical facility location model. At the first (highest) level, facilities manufacture different products, each offering a single product to the market. Second-level facilities serve as warehouses, and the third level comprises clients with demands for specific products, each exhibiting preferences based on the product's origin. The problem is to determine which depots to open and which products to allocate to them to maximize overall client satisfaction. This model is referred to as the multi-product maximal covering second-level facility location problem. The authors formulate the problem as a mixed-integer linear program and incorporate additional inequalities and constraints, as well as several metaheuristic variants, to reduce computation time.

5.3.2 Graph theoretic approaches in humanitarian logistics

In addition to other combinatorial optimization approaches, graph theory has been applied to the strategic planning of humanitarian relief distribution in disaster settings; however, only a limited number of studies exist, and these predominantly focus on using graphs as modeling tools rather than addressing theoretical aspects.

For instance, in [60], the authors proposed a multi-echelon, multi-period supply chain network design model for humanitarian logistics, solved with a graph theory-based algorithm. The model allows lateral transshipments among retailers and direct or indirect shipments from distribution centers to end-users, aiming to minimize total costs while enhancing service to evacuees.

The paper [2] tackles the challenge of managing returned products in reverse logistics, which is a critical area within supply chain management. They propose a Graph Theory and Matrix Approach (GTMA) to select the best disposition option, accounting for factors such as legislation, environmental concerns, and corporate responsibility, and illustrate the method through a case study of an Indian mobile phone manufacturer.

Table 5.2 compiles a comparative summary of the works discussed in this subsection. Notwithstanding these efforts, theoretical research on graph-theoretic modeling of multi-level supply chain networks—both broadly and in the context of post-disaster operations—remains limited. This gap highlights an open and promising direction for future investigation.

5.3.3 Evacuation planning

Evacuation planning in the aftermath of large-scale disasters has been extensively investigated in the literature, with research spanning a wide range of perspectives from traffic assignment and traffic flow propagation models to insights from human behavior science (see [6] for a comprehensive review). As highlighted in [6], most evacuation studies concentrate on private vehicle usage (commonly termed car-based evacuation), while models addressing mass transit (or bus-based) evacuation are relatively limited. The methodological approaches are equally diverse, including multi-agent–based models [42], flow-based evacuation models, and heuristic techniques (see [12] for an in-depth overview). As noted in Section 5.2, some works focus on joint optimization of shelter selection and evacuation planning [7, 8, 10, 29, 32, 34, 47, 63]. In this subsection, we will focus on the works focusing on route design in evacuation.

In [56], the authors proposed a novel approach to coordinated evacuation route planning that emphasizes guiding groups of spatially separated but socially connected individuals (e.g., families) to rendezvous at designated points along their routes before proceeding to a shelter. They formulate an integer linear programming model that minimizes the total route weight while ensuring that, once reunited, group members adjust to the speed of the slowest individual.

The authors in [19] develop an integer linear programming model that incorporates six distinct objectives, including minimizing travel distances for both primary and backup paths, reducing risks associated with routes and shelters, and optimizing the number of shelters required. They further integrate this model into a decision support tool built on geographical information systems (GIS) software.

The study in [1] addresses fair transit trip planning for emergency evacuations, with a particular emphasis on "short-notice" and "no-notice" scenarios where rapid decision-making is critical. The authors propose a model based on the *proportional fairness* concept, which seeks to balance the efficient evacuation of vulnerable populations with the equitable allocation of resources across all affected communities. This stands in contrast to maximum safety models that emphasize efficiency, often at the expense of less accessible or lower-priority groups, thereby risking resource starvation.

Table 5.2: Comparative summary of hierarchical facility location models.

Citation	Levels	Key Features / Constraints	Solution Approach	Uncertainty	Facility Type / Mode	Case Study	Notable Features
Şahin and Süral (2007) [61]	Multi-level (review)	Flow patterns, service availability, spatial configuration	Literature review	No	General HFLM	-	Foundational classification framework
Ortiz-Astorquiza et al. (2018) [58]	Multi-level (review)	Updated classification of hierarchical location models	Literature review	No	General HFLM	-	Overview of contemporary advances
Vieira et al. (2019) [70]	2	Max distance, vertex eligibility	MILP solved with CPLEX	Deterministic	Health facilities	Rio de Janeiro, Brazil	Oncological treatment units, uncapacitated median model
Ye & Xu (2015) [72]	2 (dynamic)	Time-varying demand, strategic deployment of service centers	Binary IP + GIS + Genetic Algorithm	Dynamic demand	Service facilities	-	Dynamic allocation and planning
Ghaderi et al. (2025) [30]	Multi-level + network	Multi-period budget constraints, network link optimization	MILP	Budget and network constraints	Mixed facilities + network	Urban centers	Multi-period, integrated location & network design
Baldomero-Naranjo et al. (2024) [4]	3	Product allocation, client preferences, maximal covering for multi-product distribution	MILP + metaheuristics	Deterministic	Manufacturing / Warehouses / Clients	-	Three-level multi-product distribution network

Table 5.3: Comparative summary of evacuation planning studies.

Citation	Facility / Mode	Focus / Objective	Modeling Approach	Uncertainty	Evacuation Type	Case Study	Notable Features
Nakao et al. (2025) [56]	Pedestrian groups	Coordinated routing with rendezvous points	ILP	No	Pedestrian	No	Socially connected groups, adjust to slowest member
Coutinho et al. (2012) [19]	Mixed (shelter allocation)	Multi-objective optimization (distance, risk, shelters)	ILP + GIS tool	Partial (risk)	Mixed	No	Six objectives, GIS-based DSS
Aalami et al. (2021) [1]	Transit	Fair trip planning under time pressure	Proportional fairness model	No	Bus / transit	No	Short- and no-notice scenarios, equity focus
Feng et al. (2023) [28]	Bus	Network flow planning, minimize travel + evac. time	LP + heuristic	Tested via Monte Carlo	Bus	Flood evacuation, Xingguo (China)	Two-step algorithm, outperforms greedy/genetic/approx.
Heydar et al. (2016) [35]	Transit-dependent pop.	Joint pedestrian assignment + bus routing	MIP + heuristic (SA)	No	Bus + pedestrian	No	Split-service pickup, first to integrate pedestrian + bus

The authors in [28] propose a two-step network flow planning algorithm aimed at minimizing both evacuation and computational times in disaster scenarios. The first step employs a linear programming model to optimize total bus travel time, while the second step allocates tasks uniformly across buses to approximate the minimal overall evacuation time. Monte Carlo simulations indicate that the proposed algorithm outperforms greedy, genetic, and approximation-based methods. The approach was successfully implemented in a flood evacuation case study in Xingguo, China.

In [35], the authors propose a mixed-integer programming model that integrates pedestrian guidance with heterogeneous bus routing to address long-notice evacuations of transit-dependent populations. The model incorporates a split-service feature, enabling evacuees to be assigned to multiple pickup points and buses. Given the NP-hardness of the problem, a two-stage heuristic is developed: evacuee assignment is first determined through a relaxed transportation model, followed by bus routing solved via a simulated annealing algorithm tailored for the multi-depot setting. This study is the first work that jointly optimizes pedestrian assignment and bus routing, thereby providing a practical framework for offline evacuation planning.

Table 5.3 presents a comparative summary of the works discussed in this subsection. Building on these observations, it is evident that a comprehensive graph theoretic approach to route planning in evacuation, in particular for bus-based evacuation, remains an open and interesting research issue.

5.4 Conclusions

Combinatorial optimization has become an essential tool for effective disaster management, addressing challenges across infrastructure design, shelter allocation, humanitarian logistics, and evacuation planning. This chapter has reviewed key developments in each of these areas, highlighting both methodological innovations and practical applications. Despite considerable progress, significant research gaps remain, particularly in integrating graph-theoretic optimization, uncertainty modeling, and multi-level or multi-modal considerations. Shelter layout design, joint optimization of shelter allocation and evacuation, and resilient multi-echelon logistics networks represent promising avenues for future work. Advancing these areas has the potential to enhance both the efficiency and equity of disaster response, ultimately improving the safety and well-being of affected populations. By bridging theoretical advances with practical implementation, future research can contribute to more resilient, adaptive, and socially responsive disaster management systems.

Bibliography V

[1] Soheila Aalami and Lina Kattan. “Fair transit trip planning in emergency evacuations: A combinatorial approach”. In: *Transportation Research Part C: Emerging Technologies* 122 (2021), paper no. 102760. doi: 10.1016/j.trc.2020.102760.

[2] Saurabh Agrawal, Rajesh K. Singh, and Qasim Murtaza. “Disposition decisions in reverse logistics: Graph theory and matrix approach”. In: *Journal of Cleaner Production* 137 (2016), pp. 93–104. doi: 10.1016/j.jclepro.2016.07.045.

[3] Ali Asadabadi and Elise Miller-Hooks. “Maritime port network resiliency and reliability through co-opetition”. In: *Transportation Research Part E: Logistics and Transportation Review* 137.C (2020), paper no. 101916. doi: 10.1016/j.tre.2020.101916.

[4] Marta Baldomero-Naranjo, Luisa I. Martínez-Merino, and Antonio M. Rodríguez-Chía. “On the complexity of the upgrading version of the maximal covering location problem”. In: *Networks* 83.4 (2024), pp. 627–641. doi: 10.1002/net.22207.

[5] Chandra Balijepalli and Osei Oppong. “Measuring vulnerability of road network considering the extent of serviceability of critical road links in urban areas”. In: *Journal of Transport Geography* 39 (2014), pp. 145–155. doi: 10.1016/j.jtrangeo.2014.06.025.

[6] Vedat Bayram. “Optimization models for large scale network evacuation planning and management: A literature review”. In: *Surveys in Operations Research and Management Science* 21 (2016), pp. 63–84. doi: 10.1016/j.sorms.2016.11.001.

[7] Vedat Bayram, Barbaros Ç. Tansel, and Hande Yaman. “Compromising system and user interests in shelter location and evacuation planning”. In: *Transportation Research Part B* 72 (2015), pp. 146–163. doi: 10.1016/j.trb.2014.11.010.

[8] Vedat Bayram and Hande Yaman. “Shelter location and evacuation route assignment under uncertainty: A benders decomposition approach”. In: *Transportation Science* 52.2 (2017), pp. 416–436. doi: 10.1287/trsc.2017.0762.

[9] Arup Kumar Bhattacharjee and Anirban Mukhopadhyay. “Multiobjectivization of the single-allocation hub median problem: an improved genetic algorithm-based approach”. In: *Proceedings of the Indian National Science Academy* (2025). doi: 10.1007/s43538-025-00538-4.

[10] Douglas R. Bish. “Planning for a bus-based evacuation”. In: *OR Spectrum* 33 (2011), pp. 629–654. doi: 10.1007/s00291-011-0256-1.

[11] Abtin Boostani, Fariborz Jolai, and Ali Bozorgi-Amiri. “Designing a sustainable humanitarian relief logistics model in pre- and postdisaster management”. In: *International Journal of Sustainable Transportation* 15.8 (2021), pp. 604–620. doi: 10.1080/15568318.2020.1773975.

[12] Sarah Bretschneider. *Mathematical Models for Evacuation Planning in Urban Areas*. Berlin, Heidelberg: Springer, 2012, pp. xii+193.

[13] Jianing Cao, Miaohan Zhang, Nan Pan, Yuhang Han, Jian Liu, Zhaolei He, and Zhen Ai. “Optimization of three-echelon logistics supply chain considering emergency scenarios under resilience strategy: A case study in power metering industry”. In: *Computers & Industrial Engineering* 203 (2025), paper no. 110985. doi: 10.1016/j.cie.2025.110985.

[14] Aakil M. Caunhye, Xiaofeng Nie, and Shaligram Pokharel. “Optimization models in emergency logistics”. In: *Socio-Economic Planning Sciences* 46 (2012), pp. 4–13. doi: https://doi.org/10.1016/j.seps.2011.04.004.

[15] Stephanie E. Chang, Timothy McDaniels, Jacek Mikawoz, and Karen Peterson. “Infrastructure failure interdependencies in extreme events: Power outage consequences in the 1998 Ice Storm”. In: *Natural Hazards* 41.2 (2010), pp. 337–358. doi: 10.1007/s11069-006-9039-4.

[16] Albert Y. Chen and Ting-Yi Yu. “Network based temporary facility location for the Emergency Medical Services considering the disaster induced demand and the transportation infrastructure in disaster response”. In: *Transportation Research Part B* 91 (2016), pp. 408–423. doi: 10.1016/j.trb.2016.06.004.

[17] Zhifen Chen, Xiang Chen, Qiang Li, and Jin Chen. “The temporal hierarchy of shelters: a hierarchical location model for earthquake-shelter planning”. In: *International Journal of Geographical Information Science* 27.8 (2013), pp. 1612–1630. doi: 10.1080/13658816.2013.763944.

[18] Richard L. Church and Ross A. Gerrard. “The Multi-Level Location Set Covering Model”. In: *Geographical Analysis* 35.4 (2003), pp. 281–298. doi: 10.1111/j.1538-4632.2003.tb01115.x.

[19] João Coutinho-Rodrigues, Lino Tralhão, and Luís Alçada-Almeida. “Solving a location-routing problem with a multiobjective approach: the design of urban evacuation plans”. In: *Journal of Transport Geography* 22 (2012), pp. 206–218. doi: 10.1016/j.jtrangeo.2012.01.006.

[20] Eduardo L. De Oliveira, Licìnio da Silva Portugal, and Wilson P. Junior. “Indicators of reliability and vulnerability: Similarities and differences in ranking links of a complex road system”. In: *Transportation Research Part A: Policy and Practice* 88 (2016), pp. 195–208. doi: 10.1016/j.tra.2016.04.004.

[21] Mohammad Saied Dehghani, Gerardo W. Flintsch, and Sue McNeil. “Impact of road conditions and disruption uncertainties on network vulnerability”. In: *Journal of Infrastructure Systems* 20.1 (2014), paper no. 04014015. doi: 10.1061/(ASCE)IS.1943-555X.0000205.

[22] Hande Demirel, Mert Kompil, and Françoise Nemry. “A framework to analyze the vulnerability of European road networks due to Sea-Level Rise (SLR) and sea storm surges”. In: *Transportation Research Part A: Policy and Practice* 81 (2015), pp. 62–76. doi: 10.1016/j.tra.2015.05.002.

[23] Zehranaz Dönmez, Bahar Y. Kara, Özlem Karsu, and Francisco Saldanha-da-Gama. “Humanitarian facility location under uncertainty: Critical review and future prospects”. In: *Omega* 102 (2021), paper no. 102393. doi: 10.1016/j.omega.2021.102393.

[24] Zvi Drezner and George O. Wesolowsky. “Covering part of a planar network”. In: *Journal of Mathematical Modelling of Systems* 20.3 (2014), pp. 245–263. doi: 10.1007/s11067-014-9263-5.

[25] Levent Eriskin and Mumtaz Karatas. “Applying robust optimization to the shelter location-allocation problem: a case study for Istanbul”. In: *Annals of Operations Research* 339 (2024), pp. 1589–1635. doi: 10.1007/s10479-022-04627-1.

[26] Reza Zanjirani Farahani, Masoud Hekmatfar, Behnam Fahimnia, and Narges Kazemzadeh. “Hierarchical facility location problem: Models, classifications, techniques, and applications”. In: *Computers & Industrial Engineering* 68 (2014), pp. 104–117. doi: 10.1016/j.cie.2013.12.005.

[27] Reza Faturechi and Elise Miller-Hooks. “Measuring the performance of transportation infrastructure systems in disasters: A comprehensive review”. In: *Journal of Infrastructure Systems* 21.1 (2015), paper no. 04014025. doi: 10.1061/(ASCE)IS.1943-555X.0000212.

[28] Yuanyuan Feng, Yi Cao, Shuanghua Yang, Lili Yang, and Tangjian Wei. “A two-step sub-optimal algorithm for bus evacuation planning”. In: *Operational Research* 23.2 (2023), paper no. 36. doi: 10.1007/s12351-023-00781-x.

[29] Melissa Gama, Bruno Flippe Santos, and Maria Paola Scaparra. “A multi-period location-allocation model with evacuation orders for flood disasters”. In: *EURO Journal on Computational Optimization* 4.3-4 (2016), pp. 299–323. doi: 10.1007/s13675-015-0058-3.

[30] Abdolsalam Ghaderi, Azade Modarres, and Zahra Hosseinzadeh Bandbon. “A Hierarchical Multi-Period Approach to Integrated Facility Location and Network Design”. In: *Advances in Industrial Engineering* 59.1 (2025), pp. 23–41. doi: 10.22059/aie.2024.379397.1901.

[31] Javid Ghahremani-Nahr, Hamed Nozari, and Agnieszka Szmelter-Jarosz. “Designing a humanitarian relief logistics network considering the cost of deprivation using a robust-fuzzy-probabilistic planning method”. In: *Journal of International Humanitarian Action* 9.19 (2024), pp. 1–16. doi: 10.1186/s41018-024-00163-8.

[32] Marc Goerigk, Kaouthar Deghdak, and Philipp Hessler. “A comprehensive evacuation planning model and genetic solution algorithm”. In: *Transportation Research Part E* 71 (2014), pp. 82–97. doi: 10.1016/j.tre.2014.08.007.

[33] Amir Golroo, Abdolreza Mohaymany, and Mahmoud Mesbah. “Reliability based investment prioritization in transportation networks”. In: *Proceedings of the 89th Annual Meeting of the Transportation Research Board*. Washington, DC: Transportation Research Board, 2010. url: https://www.academia.edu/523720/Reliability_Based_Investment_Prioritization_in_Transportation_Networks (visited on 09/26/2025).

[34] Philipp Hessler and Horst W. Hamacher. “Sink locations to find optimal shelters in evacuation planning”. In: *EURO Journal on Computational Optimization* 4.3-4 (2016), pp. 324–347. doi: 10.1007/s13675-015-0054-7.

[35] Mojtaba Heydar, Jie Yu, Yue Liu, and Matthew E. H. Petering. “Strategic evacuation planning with pedestrian guidance and bus routing: a mixed integer programming model and heuristic solution”. In: *Journal of Advanced Transportation* 50.7 (2016), pp. 1314–1335. doi: 10.1002/atr.1403.

[36] Mark W. Horner and Joni A. Downs. “Optimizing hurricane disaster relief goods distribution: model development and application with respect to planning strategies”. In: *Disasters* 34.3 (2010), pp. 821–844. doi: 10.1111/j.1467-7717.2010.01171.x.

[37] Bramka Arga Jafino, Jan H. Kwakkel, and Alexander Verbraeck. “Transport network criticality metrics: a comparative analysis and a guideline for selection”. In: *Transport Reviews* 39.6 (2019), pp. 775–797. doi: 10.1080/01441647.2019.1703843.

[38] Erik Jenelius, Tom Petersen, and Lars-Göran Mattsson. “Importance and exposure in road network vulnerability analysis”. In: *Transportation Research Part A: Policy and Practice* 40.7 (2006), pp. 537–560. doi: 10.1016/j.tra.2005.11.003.

[39] Hongzhong Jia, Fernando Ordóñez, and Maged M. Dessouky. “Solution approaches for facility location of medical supplies for large-scale emergencies”. In: *Computers & Industrial Engineering* 52 (2007), pp. 257–276. doi: 10.1016/j.cie.2006.12.007.

[40] Afshin Kamyabniya, Antoine Sauré, F. Sibel Salman, Noureddine Bénichou, and Jonathan Patrick. "Optimization models for disaster response operations: a literature review". In: *OR Spectrum* 46.3 (2024), pp. 737–783. doi: 10.1007/s00291-024-00750-6.

[41] Özlem Karsu and Bahar Yetiş Kara. "The refugee camp management: a general framework and a unifying decision-making model". In: *Journal of Humanitarian Logistics and Supply Chain Management* 19.2 (2019), pp. 131–150. doi: 10.1108/JHLSCM-01-2018-0007.

[42] Navroop Kaur and Harjot Kaur. "A multi-agent based evacuation planning for disaster management: A narrative review". In: *Archives of Computational Methods in Engineering* 29 (2022), pp. 4085–4113. doi: 10.1007/s11831-022-09729-4.

[43] Ali Kermanshah and Sybil Derrible. "Robustness of road systems to extreme flooding: Using elements of GIS, travel demand, and network science". In: *Natural Hazards* 84.1 (2016), pp. 367–388. doi: 10.1007/s11069-016-2678-1.

[44] Fırat Kilci, Bahar Yetiş Kara, and Burçin Bozkaya. "Locating temporary shelter areas after an earthquake: A case for Turkey". In: *European Journal of Operational Research* 243.1 (2015), pp. 323–332. doi: 10.1016/j.ejor.2014.11.035.

[45] Ömer Burak Kinay, Bahar Yetis Kara, Francisco Saldanha-da-Gama, and Isabel Correia. "Modeling the shelter site location problem using chance constraints: A case study for Istanbul". In: *European Journal of Operational Research* 270.1 (2018), pp. 132–145. doi: 10.1016/j.ejor.2018.03.006.

[46] Fumitaka Kurauchi, Nobuhiro Uno, Agachai Sumalee, and Yumiko Seto. "Network evaluation based on connectivity vulnerability". In: *Proceedings of the Eastern Asia Society for Transportation Studies*. Vol. 7. 2009, pp. 348–348. doi: 10.1007/978-1-4419-0820-9_31.

[47] Anna C. Y. Li, Linda Nozick, Ningxiong Xu, and Rachel Davidson. "Shelter location and transportation planning under hurricane conditions". In: *Transportation Research Part E* 48.4 (2012), pp. 715–729. doi: 10.1016/j.tre.2011.12.004.

[48] Huiyong Li, Laijun Zhao, Rongbing Huang, and Qingmi Hu. "Hierarchical earthquake shelter planning in urban areas: A case for Shanghai in China". In: *International Journal of Disaster Risk Reduction* 22 (2017), pp. 431–446. doi: 10.1016/j.ijdrr.2017.01.007.

[49] Yen-Hung Lin, Rajan Batta, Peter A. Rogerson, Alan Blatt, and Marie Flanigan. "Location of temporary depots to facilitate relief operations after an earthquake". In: *Socio-Economic Planning Sciences* 46 (2012), pp. 112–123. doi: 10.1016/j.seps.2012.01.001.

[50] London Resilience Forum. *Multi-Agency Shelter Framework*. 2018. url: https://www.london.gov.uk/sites/default/files/london_mass_shelter_framework_v4_3_october_2018.pdf (visited on 09/10/2025).

[51] Cristina Losada, Maria Paola Scaparra, and Jeremiah R. O'Hanley. "Optimizing system resilience: A facility protection model with recovery time". In: *European Journal of Operational Research* 217.3 (2012), pp. 519–530. doi: 10.1016/j.ejor.2011.09.044.

[52] Lars-Göran Mattsson and Erik Jenelius. "Vulnerability and resilience of transport systems – A discussion of recent research". In: *Transportation Research Part A: Policy and Practice* 81 (2015), pp. 16–34. doi: 10.1016/j.tra.2015.06.002.

[53] Sabyasachee Mishra, Timothy F. Welch, and Mithilesh Kumar Jha. "Performance indicators for public transit connectivity in multi-modal transportation networks". In: *Transportation Research Part A: Policy and Practice* 46.7 (2012), pp. 1066–1085. doi: 10.1016/j.tra.2012.04.006.

[54] Ahmad Mohamadi and Saeed Yaghoubi. "A bi-objective stochastic model for emergency medical services network design with backup services for disasters under disruptions: An earthquake case study". In: *International Journal of Disaster Risk Reduction* 23 (2017), pp. 204–217. doi: 10.1016/j.ijdrr.2017.05.003.

[55] Mehrnaz Najafi, Ali Ghodratnama, and Hamid Reza Pasandideh. "The optimization of a multi-product three-echelon supply chain". In: *International Journal of Supply and Operations Management* 4.3 (2017), pp. 229–247. doi: 10.22034/2017.3.04.

[56] Riku Nakao, Akihito Hiromori, Hamada Rizk, and Hirozumi Yamaguchi. "Optimizing coordinated evacuation route planning based on linear programming". In: *IEEE International Conference on Smart Computing*. 2025, pp. 414–419. doi: 10.1109/SMARTCOMP65954.2025.00118.

[57] Mehmet Kursat Oksuz and Sule Itir Satoglu. "A two-stage stochastic model for location planning of temporary medical centers for disaster response". In: *International Journal of Disaster Risk Reduction* 44 (2020), paper no. 101426. doi: 10.1016/j.ijdrr.2019.101426.

[58] Camilo Ortiz-Astorquiza, Ivan Contreras, and Gilbert Laporte. "Multi-level facility location problems". In: *European Journal of Operational Research* 267.3 (2018), pp. 791–805. doi: 10.1016/j.ejor.2017.10.019.

[59] Vandana Patidar, Samuel Labi, Kumares C. Sinha, and Paul Thompson. *Multi-objective optimization for bridge management systems*. Tech. rep. NCHRP Report 590. Transportation Research Board, 2007. url: https://www.trb.org/publications/nchrp/nchrp_rpt_590.pdf (visited on 09/19/2025).

[60] Masoud Rabbani, Ali Sabbaghnia, Mahdi Mobini, and Jafar Razmi. "A graph theory-based algorithm for a multi-echelon multi-period responsive supply chain network design with lateral-transshipments". In: *Operational Research* 20.4 (2020), pp. 2497–2517. doi: 10.1007/s12351-018-0425-y.

[61] Güvenç Şahin and Haldun Süral. "A review of hierarchical facility location models". In: *Computers & Operations Research* 34.8 (2007), pp. 2310–2331. doi: 10.1016/j.cor.2005.09.005.

[62] Darren M. Scott, David C. Novak, Lisa Aultman-Hall, and Feng Guo. "Network robustness index: A new method for identifying critical links and evaluating the performance of transportation networks". In: *Journal of Transport Geography* 14.3 (2006), pp. 215–227. doi: 10.1016/j.jtrangeo.2005.10.003.

[63] Shahrooz Shahparvari, Prem Chhetri, Babak Abbasi, and Ahmad Abareshi. "Enhancing emergency evacuation response of late evacuees: Revisiting the case of Australian Black Saturday bushfire". In: *Transportation Research Part E* 93 (2016), pp. 148–176. doi: https://doi.org/10.1016/j.tre.2016.05.010.

[64] Bahar Shahverdi, Mersedeh Tariverdi, and Elise Miller-Hooks. "Assessing hospital system resilience to disaster events involving physical damage and demand surge". In: *Socio-Economic Planning Sciences* 70 (2019), paper no. 100729. doi: 10.1016/j.seps.2019.07.005.

[65] Bhuvnesh Sharma, M. Ramkumar, Nachiappan Subramanian, and Bharat Malhotra. "Dynamic temporary blood facility location-allocation during and post-disaster periods". In: *Annals of Operations Research* 283 (2019), pp. 705–736. doi: 10.1007/s10479-017-2680-3.

[66] Douglas R. Shier. "On algorithms for finding the *k* shortest paths in a network". In: *Networks* 9.3 (1979), pp. 195–214. doi: 10.1002/net.3230090302.

[67] Maaike Snelder, Henk J. van Zuylen, and Lambertus H. Immers. "A framework for robustness analysis of road networks for short term variations in supply". In: *Transportation Research Part A: Policy and Practice* 46.5 (2012), pp. 828–842. doi: 10.1016/j.tra.2012.02.007.

[68] James. L. Sullivan, David. C. Novak, Lisa. Aultman-Hall, and Darren. M. Scott. "Identifying critical road segments and measuring system-wide robustness in transportation networks with isolating links: A link-based capacity-reduction approach". In: *Transportation Research Part A: Policy and Practice* 44.5 (2010), pp. 323–336. doi: 10.1016/j.tra.2010.02.003.

[69] United Nations Refugee Agency. *Camp planning standards (planned settlements)*. 2017. url: https://emergency.unhcr.org/ (visited on 09/10/2025).

[70] IIsabella F. G. Vieira, Matheus Ferreira de Barros, and Allan Cormack. "Hierarchical facility location model for allocating cancer treatment units in interior of Rio de Janeiro". In: *Production* 29 (2019), paper no. e20190026. doi: 10.1590/0103-6513.20190026.

[71] Jinghai Xu, Xiaozhe Yin, Dingchao Chen, Jiwen An, and Gaozong Nie. "Multi-criteria location model of earthquake evacuation shelters to aid in urban planning". In: *International Journal of Disaster Risk Reduction* 20 (2016), pp. 51–62. doi: 10.1016/j.ijdrr.2016.10.009.

[72] Wei Ye and Zhitao Xu. "Dynamic facility location and allocation optimization for sustainable product-service delivery". In: *Sustainability* 17.17 (2025), paper no. 8000. doi: 10.3390/su17178000.

[73] Naeme Zarrinpoor, Mohammad Saber Fallahnezhad, and Mir Saman Pishvaee. "Design of a reliable hierarchical location-allocation model under disruptions for health service networks: A two-stage robust approach". In: *Computers & Industrial Engineering* 109 (2017), pp. 130–150. doi: 10.1016/j.cie.2017.04.036.

[74] Xiujuan Zhao, Wei Xu, Yunjia Ma, and Fuyu Hu. "Scenario-based multi-objective optimum allocation model for earthquake emergency shelters using a modified particle swarm optimization algorithm: A case study in Chaoyang district, Beijing, China". In: *PLOS One* 10.12 (2015), paper no. e0144455. doi: 10.1371/journal.pone.0144455.

[75] Yang Zhou, Zhixiang Fang, Jean Claude Thill, Qingquan Li, and Yuguang Li. "Functionally critical locations in an urban transportation network: Identification and space-time analysis using taxi trajectories". In: *Computers, Environment and Urban Systems* 52 (2015), pp. 34–47. doi: 10.1016/j.compenvurbsys.2015.03.001.

Chapter VI

15-Minute City

This chapter investigates the 15-minute city, an urban planning paradigm that ensures essential services are accessible within a short walk or bike ride. It surveys computational and mathematical approaches to operationalize the concept, including graph-theoretic and grid tessellation models, optimization methods such as mixed-integer linear programming, and interactive tools for accessibility analysis. The chapter further reviews commonly used indices for assessing accessibility, diversity, and walkability. Consequently, it highlights how combinatorial optimization and data-driven planning can address spatial inequalities and foster sustainable, equitable, and locally oriented urban development.

Keywords: 15-minute city, facility location, walkability, accessibility, sustainability.

VI

15-Minute City

Magda Dettlaff, University of Gdańsk, Poland
Magdalena Lemańska, Gdańsk University of Technology, Poland
María José Pereira-Sáez, University of A Coruña, Spain
María José Souto-Salorio, University of A Coruña, Spain

The concept of *15-minute city*, introduced and promoted by urbanist Carlos Moreno [11], is a paradigm of modern urban planning that seeks to provide residents with access to all essential urban services within a 15-minute walk or bike ride from their home. Services typically include education, healthcare, employment, food, culture, and leisure. This model emphasizes localized living, supports the environment (reducing dependency on motorized transport), improves the overall quality of urban life, and promotes sustainable urban development.

The main objective of this vision is to create self-sufficient neighborhoods where people can live, work, shop, and socialize without long commutes. Such decentralization also helps distribute resources more equitably across cities. However, to put this idea into practice, especially in large metropolitan areas, urban planners require rigorous mathematical and computational models to evaluate service accessibility and optimize infrastructure development.

This chapter provides a comprehensive review of the computational and mathematical frameworks developed to operationalize and assess this vision. Specifically, it surveys graph-theoretic and grid tessellation models for representing urban networks and quantifying service accessibility, as well as optimization approaches based on mixed-integer linear programming for the strategic allocation of new amenities. The study further analyzes interactive platforms that facilitate the visualization and evaluation of accessibility at multiple spatial scales, along with indices designed to capture accessibility, diversity, and walkability in urban environments. By synthesizing these methodological perspectives, the report underscores the critical role of combinatorial optimization and computational modeling in advancing the practical implementation of the 15-minute city, while highlighting their relevance for addressing spatial inequalities and supporting sustainable urban transformation.

6.1 15-MINUTE CITY CONCEPT

The literature presents a broad range of papers on the 15-minute city concept. Most of them present an urban planning or sociological approach to this concept. Quite a lot of research has contributed greatly to the literature of measuring city performance from a 15-minute city perspective. In [13] Papadopoulos et al. provide a literature survey and recommendations for a more holistic compliance assessment. We, in turn, will focus on research that treats the topic from the point of view of combinatorial optimization. We present several approaches to the topic. We present several approaches to the topic. We begin with graph-theory-based and grid-tessellation models, followed by models formulated through linear programming. Next, we discuss interactive tools that demonstrate how far specific regions are from achieving the 15-minute city model. Finally, we address accessibility aspects.

6.1.1 Graph and grid tessellation models

First, we review papers that primarily address graph-theoretic and grid-tessellation approaches to the 15-minute city. According to the graph theory models, if a graph $G = (V, E)$ with a set V of vertices and a set E of edges represents the city, then the vertices of G are usually intersections of the roads and the edges are the passable streets connecting these intersections. The lengths of these edges are proportional to the travel time of the pedestrians, and since the pedestrians can move along the streets in both directions, the graph is undirected. This model is considered for example in [3], where the physical length of each street (section between two intersections) is determined on the basis of geographical data, in this case, coming from Open Street Maps (OSM). To obtain data on streets, OSMnx Python was used. The physical length of the street was then converted into pedestrian travel time assuming an average walking speed 1.22 m/s.

If service $f_i \in V$ of a given type i is placed in a given vertex, then a shortest path from which this service is reachable for each vertex of the graph and the corresponding travel time can be found. Applying algorithms of finding the shortest paths to all vertices, we can find all the vertices in the graph that are in the a distance of at most 15 minutes from a service f_i. So, we can define as C^i, the set of vertices (crossing of the roads), which forms a 15–minute city according to the services of a given type i; i.e. C^i is set of all vertices from which we can reach at least one service of type i at time shorter than 15–minutes. Then, considering a subgraph G_i of an urban graph G induced by the set C^i, we can study for example a connectedness of G_i to immediately identify places where an intervention is necessary to connect, for example, two parts of the city with some service.

We can consider also the case of different services which must be available in less than 15 minutes. If there are K types of services, the set of 15–minute vertices has a form $C = \bigcap_{i=1}^{K} C^i$. Thus, if G_C is a subgraph of G induced by the vertices of C, then 15– minute city graph is a subgraph G_C of an urban graph G. Despite the simplicity of this approach, we can analyze with it some urban aspects, for example, analyze the connectedness of the G_C graph.

But the set C depends on many parameters such as the time of the travel t, the matrix of services s, and the travel speed v; 15-minute city graph for cyclists will be different than for pedestrians:

$$G_C = G_C(C, E_C; t, s, v) \subseteq G.$$

This means that there is no single 15-minute city graph; rather, each set of services defines a distinct possible city. Additionally, changing the parameter t, we can obtain a t-minute city graph for any t. In [3], G_C is in the form: $G_C = G_C(C, E_C;$ 15 min, 'farmacies', 'post-offices', 'supermarkets'), 1.22 m/s).

In [3], the authors propose the 15-minute city (15MC) index, designed both to characterize the concept and to facilitate comparisons across cities or between different areas of the same city:

$$\gamma(r, x_0; t, s, v) \equiv |C(r, x_0; t, s, v)|/|E|,$$

where $|C(r, x_0; t, s, v)|$ denotes the number of vertices within a radius r from x_0, which belong to the 15-minute city, defined with respect to services s at travel time t and travel speed v. In other words, it represents the number of locations from which one can access all essential services within 15 minutes of point x_0.

In [5], a slightly different yet related approach is proposed: the pedestrian network is represented as a graph, where the vertices are intersection points of paths and the edges are segments of those paths, with attributes, including the time cost. In this graph, all available walking routes in public space are mapped, including sidewalks, crossing paths, and pedestrian zones, particularly those that allow the passage through green areas. The network also includes virtual paths assuming that users, in the absence of a dedicated path, may choose to walk on the side of the road or pass near intersections. For each link (edge) in the pedestrian network, was counted the value of 'cost', which is the ratio of the length of the link to the speed of the pedestrian, estimated as 3 km/h. Calculation of the cost, expressed in minutes, also takes into account the 'delay factor' (DF) at the crossings pedestrian crossing, which is slightly different depending on whether the crossing are with or without traffic lights:

$$\text{Travel time (min)} = (\text{Length(km)} / (3\ \text{km/h})) \cdot 60 + \text{DF(min)}.$$

Generally, edge weights on the graph are derived through GIS analysis and are expressed in minutes, accounting for the time required to traverse each segment of the pedestrian network as well as potential delays at street crossings. This formula was used in a study of the Cittadella district in Parma, Italy. The authors of [5] used it to determine the area which can be reached on foot within 15 minutes, which they defined as 'urban nodes' well equipped with the necessary services such as supermarkets, grocery stores, bars, pharmacies and banks. Then, the calculated area was compared with the distribution population of that neighborhood in order to examine the proportion of the population of the 15-minute city.

The other graph theory approach is presented in [6]. In that work, the authors report on a project carried out in Denmark's North Jutland region, with the objective of mapping the coverage and quality of mobile broadband networks across four operators. Data collection was carried out over a six-month period by two vehicles equipped with specialized measurement instruments that traversed the region. The main theme of this paper is the description of how the driving campaigns were designed. One of the main challenges was designing the driving routes to ensure that mobile coverage was measured on 100 percent of the roads in the study area —a project requirement— because of the problem's combinatorial nature. The route optimization problem faced in this project is formally defined using a connected road graph $G = (V, E)$ in a cluster C, where a route $RC = \{r_0, r_1, \ldots, r_{n+m}\}$ traversing all edges in $EC = \{e_0, e_1, \ldots, e_n\}$ has to be calculated minimizing its total length. The route must cover all edges of the road graph, remain connected, and start and end at the same vertex.

A general survey of works related to the applications of graphs in smart cities such as modeling smart cities, security, anomaly detection, transport, logistics, together with future research directions is presented in [15]. Currently, today's infrastructure provides important services including energy, water, mobility, manufactured goods, and healthcare. These infrastructures are complex, large, and interconnected. Therefore, a model that takes into account the interdependencies between networks must be created to understand the behavior of complex systems. The graph theory approach serves as the basis for many studies of network-based systems, such as power grids, water networks, supply chains, transportation systems, and health systems.

Also in [1] there is a more general approach to graph models; different graphs models for urban layout representations are compared. The paper first reviews common spatial network models and their key features, then demonstrates their application in the city of Alicante.

In [10], there is an interesting graph theory algorithmic approach to the problem of 15-minute city. The aim of the study is to develop an algorithm capable of identifying areas of the city that can be classified as a 15–minute city. This algorithm is intended to determine which locations allow residents to reach all necessary services within 15 minutes from their homes. The proposed solution takes into account different types of graphs and locations of services.

In general, Dijkstra's standard algorithm is used to find the shortest path from a single source vertex to all other vertices in a graph with non-negative weights. But in [10], a modified Dijkstra algorithm is applied to the 15-minute ciy problem. The modification is done in the following way:

- Search from the service sources: Instead of running the algorithm from each vertex in the graph, the modified Dijkstra algorithm is run multiple times, using the location of individual services or service groups as source vertices. It is more efficient, as the number of service locations is usually significantly less than the total number of vertices in the graph.

- Stop searching when the time threshold is reached: The main modification is to stop the algorithm when the distance (travel time) to the considered vertex exceeds a set time threshold t (e.g. 15 minutes).
- Identification of reachable vertices: For each execution of the algorithm from a type i service location, modified Dijkstra's algorithm denotes all vertices to which time less than or equal to t as achievable for that type of service.

In the complexity analysis, the authors consider a case in which the area of the city modeled as a graph (the whole graph G) is larger than the area which actually qualifies as a 15-minute city (to contrast it with the case when the whole city is a 15-minute city). In this scenario, the actual complexity of the algorithm depends on the subgraph size that is achievable within t minutes of the considered sources (location of services) and not on the size of the entire city.

The work in [10] centers on designing a versatile, adaptive, and efficient algorithm capable of identifying urban areas that meet the criteria of a 15-minute city. The author applies this approach to the cases studied in [3], [5], and [12], and evaluates its performance against the scenarios and methodologies proposed in those papers.

Another frequently used method of modelling the 15-minute city is the grid tessellation method. According to this method, the map is divided into a grid of 'cells', where each cell is treated as a separate area. Then the calculation of the 15-minute city is applied independently to the each of these areas. Most often, the area is divided into squares (e.g. 100m x 100m, 250m x 250m, 500m x 500m) or hexagons. Often hexagons are preferred, because they donnot have corner neighbors and that gives a better spatial continuity.

For example, the 34 Minutes Istanbul project [9], developed by the Istanbul Metropolitan Municipality, uses 2021-2022 data to map out which daily needs residents can access by walking within specific areas. The map divides Istanbul into hexagonal zones (at district and neighborhood scales), each representing a 34-minute living area, marked by a dashed black line showing the maximum round-trip distance within 34 minutes. Each hexagon is scored from 0 to 100 across categories like housing, work, daily needs, cultural activities, training, health, transportation, and leisure. Blue hexagons indicate easier access to daily needs within 34 minutes, while red hexagons signify more challenging access (see Figure 6.1).

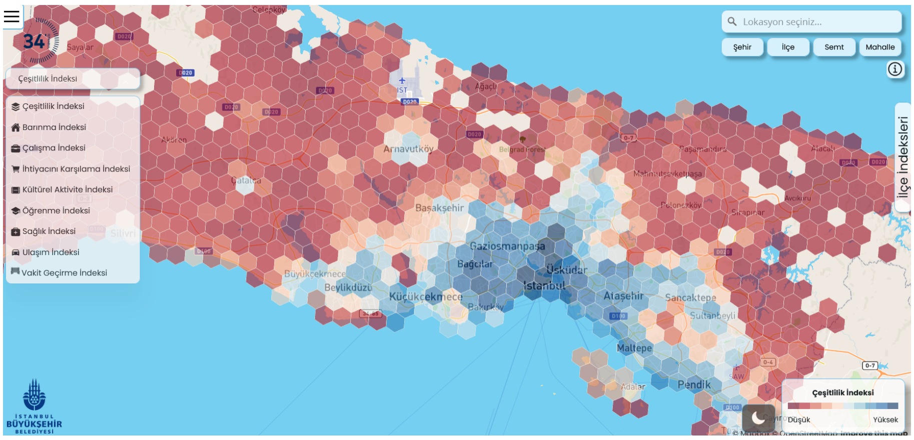

Figure 6.1: Interface of the 34 Minutes Istanbul project.

Each hexagonal cell may be examined separately with respect to service availability, population density, and other attributes. Conceptually, this resembles superimposing a grid on the map to reveal conditions across different parts of the city.

In [18], for example, the data points were organised into hexagonal grid units measuring 100m × 100m each. The average inflow distances for a grid are calculated as AID= (sum the inflow distances for each visitor from 1 to n) / (number of unique visitors to the grid), where n is the total number of inflow events recorded for the grid. The average outflow distances (AOD) is calculated based on the average travel distance for all residents of each grid. The authors also use a method that quantifies the average weighted distance of movements from specific origins using a formula. The article focuses on Hamilton (New Zealand), addressing issues of car dependency and urban sprawl, and introduces a methodology that can be adapted to other cities. A first step involving a spatial analysis was to identify 'liveable areas' based on walkable access to essential services within specific districts; the second one includes a mobility analysis that investigates residents' travel patterns, focusing particularly on their preference for 'local living'.

Very often special types of indexes are introduced and counted separately for every cell of the map. For example in [12], the NEXI index was defined; the determination of this index is based on the measurement of the level of the local proximity to services, accessible on foot, in accordance with the assumptions concept of a 15-minute city. This index aims to identify areas that already meet these criteria. NEXI is counted for any cell of a regular hexagonal covered a given area and then presented in an interactive map where every cell is colored depending of the value of the index. In [12] two main types of the index NEXI are given:

- NEXI-Minutes: For every service category and every area (hexagonal cell) assigns a time value that is the average of the walking time to the nearest service point of this category, at the assumed walking speed of 5 km/h.
- NEXI-Global: Measures overall proximity to all service categories on a scale from 0 to 100, where 0 means that none of the service categories is accessible within a 30-minute walk, and 100 means that all service categories are available within a 15-minute walk.

In [12], grid tessellation is connected with graph theory. The road network is modeled using a graph: intersections are vertices, and road sections are edges. The NEXI algorithm calculates the time it takes to walk to the nearest POI (Points of Interest) from each node (vertex) of the road network. This is done by finding the shortest path in graphs, for this purpose graph algorithms are used. After calculating the time to reach the nearest services for each node road network, this information is aggregated into grid cells hexagonal system. The NEXI index is calculated for each hexagonal cell by averaging proximity levels (reach times) nodes in that cell. It results of the calculated NEXI index are then visualized on the interactive map, where each cell of the hexagonal grid is thematically colored depending on the index value (see Figure 6.2).

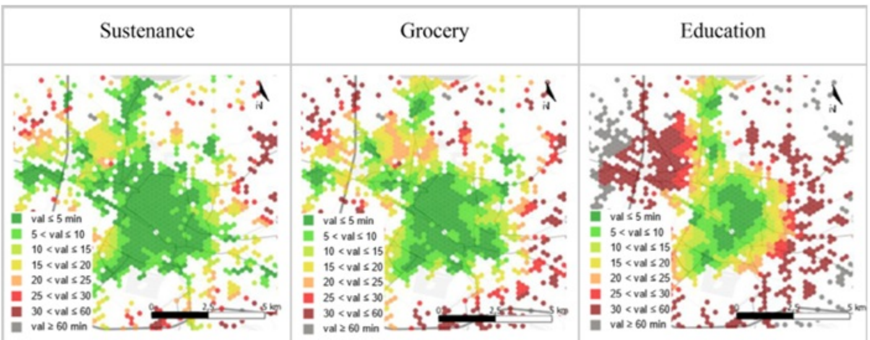

Figure 6.2: NEXI-Minutes index - Categories comparison - Ferrara [12].

6.1.2 Models based on linear programming

The idea of designing walkable cities has gained momentum because of its positive effects on public health, the economy, and environmental sustainability. However, outdated zoning rules and long-term underinvestment have led to uneven access to walkable spaces, creating social and spatial inequalities.

Huang and Khalil, in [8], formulate Walkability Optimization as the combinatorial optimization problem of selecting the locations of new amenities to maximally improve residents' access to basic necessities. The goal is to determine where new amenities-such as grocery stores, schools, or restaurants-should be added so that residents can reach them on foot more easily. This requires considering the distribution of existing facilities and ensuring multiple choices for certain services (e.g., different restaurants).

They approach the challenge of Walkability Optimization using methods from combinatorial optimization and develop models based on Mixed-Integer Linear Programming (MILP) and Constraint Programming (CP). Next, it is demonstrated that in specific cases, the problem's objective function is submodular, which supports the use of an efficient greedy heuristic. Moreover, it is shown that decision version of Walkability Optimization is NP-Complete in general.

This approach was tested through a case study of 31 underserved neighborhoods in Toronto in Canada. Results show that while MILP produces the best outcomes in most cases, it struggles to scale with larger networks. The greedy algorithm, on the other hand, performs well at scale and yields high-quality solutions.

The empirical results suggest that neighborhoods currently lacking walkability hold significant potential for transformation into pedestrian-friendly areas through strategic placement of new amenities. For instance, adding just three more grocery stores, schools, and restaurants can raise the "WalkScore" by over 50 points (on a 100-point scale) in four neighborhoods, while also reducing walking times to under 10 minutes for 75% of households across all types of amenities.

On the other side, in [17], the Location Set Covering Problem optimization model is employed to analyze the resources required to achieve full coverage of 15-minute accessibility and the knee point detection algorithm to assess a city's 15-minute city potential. The LSCP optimization model enables us to identify the best locations for new facilities to ensure that all residents can reach this category of facilities within 15 minutes of walking while minimizing the number of new facilities. So, using the Location Set Covering Problem optimization model, they estimate the resources needed for full accessibility coverage, while the knee point detection algorithm identifies the inherent potential of each city. The authors applied this approach to 23 major Chinese cities and reveal notable disparities: current levels of 15-minute city implementation often diverge from the underlying potential. Critical factors include the alignment of facility locations with population centers and the density of residents in peripheral areas. Interestingly, reducing planned facility construction by up to two-thirds results in only minor accessibility losses, underscoring the value of customized data-driven strategies. By prioritizing efficiency and avoiding underutilized infrastructure, this method highlights pathways to maximize the benefits of the 15-minute city while supporting sustainable and equitable urban futures.

In [2], Arslan and Laporte examine how the 15-minute city model can be translated into practice through the application of operations research methods. Whereas much of the existing literature has focused on defining and measuring accessibility within proximity-based urban systems, their study emphasizes operational strategies and optimization-based approaches for implementing and managing such environments. The authors review the relevant literature on operations research, highlight intersections with urban planning objectives, and outline a research agenda that prioritizes facility location planning, mobility systems, shared transportation solutions, and integrated governance mechanisms. Analyzing relevant papers, in [2], it is concluded that interest in the 15-minute city is increasing. It is also concluded that most of the journals publishing on the 15-minute city focus on urban planning, sustainability, or transportation policy, and there is little representation from core operations research journals.

6.1.3 Interactive tools

Cities organized around proximity have gained significant attention in recent years. Among these, the idea of the "15-minute city" has introduced a vision in which essential services are located within easy reach of residents. While this concept has proven valuable in sparking debate about urban reorganization, it is not universally applicable, and its definition itself raises certain issues.

In [4], the practicality and viability of the 15-minute city model is examined across a global range of urban environments. It is quantified how closely cities align with the 15-minute ideal by assessing travel times to key services and resources. The analysis shows considerable variation in accessibility both within individual cities and across different regions, with population density emerging as a decisive factor. To make these findings widely accessible, Bruno, Monteiro Melo, Campanelli et al. offer an online platform — that can be reached from the link https://whatif.sonycsl.it/15mincity/ — that allows users to explore accessibility scores for nearly every city worldwide. Figure 6.3 illustrates the interface of the application. The interactive map also allows users to check individual cities (see Figure 6.4).

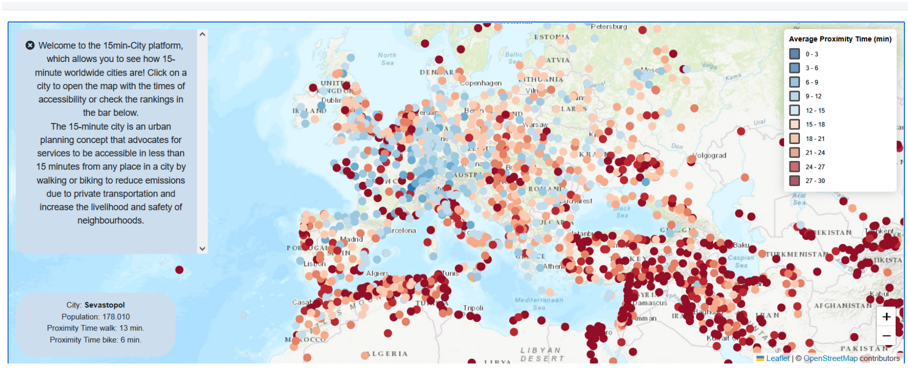

Figure 6.3: Interface of an online platform

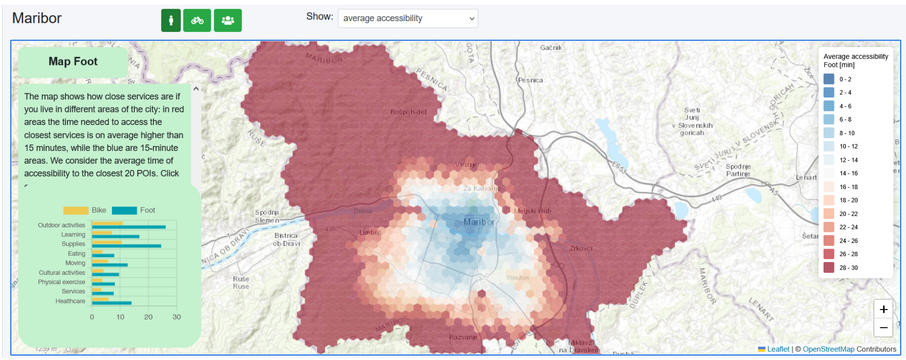

Figure 6.4: Interface of an online platform for the Maribor

Uneven distribution of accessibility contributes to urban inequality. To address this, in [4], they simulate scenarios where resources and services are redistributed more evenly, both under the constraint of existing resources and under the assumption of unlimited availability. Presented results reveal substantial differences among cities in the number of additional services required to achieve the 15-minute city standard. Ultimately, it is argued that the proximity-based approach must be adapted to accommodate diverse population densities.

Moreover, incorporating socio-economic and cultural dimensions will be essential to move from a purely time-focused framework toward one grounded in broader values.

To collect data to make analysis, we can use many online tools. One example is the SmartHubs Accessibility Tool presented in [16]. It is a user-friendly planning tool that is designed to assist planners and practitioners in developing and testing mobility hub scenarios. The tool aims to measure POI accessibility using various modes of transportation (walking, cycling, e-scooters, public transport) to and from mobility hubs. For each mean of transportation, it is possible to define additional requirements, for example maximum walking travel time in minutes and walking speed. In addition, the tool allows users to choose points of interest (POIs) relevant to them, including restaurants, cafés, bars, healthcare facilities, services, supermarkets, entertainment venues, and educational institutions. In the final step, users place a marker at the chosen location and run the analysis. The tool highlights the resulting area on the map, provides aggregated results in a table, and allows the geospatial data to be downloaded. Figure 6.5 shows the results of the accessibility analysis to restaurants, cafés, bars, healthcare facilities, and supermarkets within a 15-minute walk from the Faculty of Electrical Engineering and Computer Science, University of Maribor. Although the analysis that the tool performs is relatively simple, the tool stands out for its automation and user-friendliness, eliminating the need for GIS software expertise.

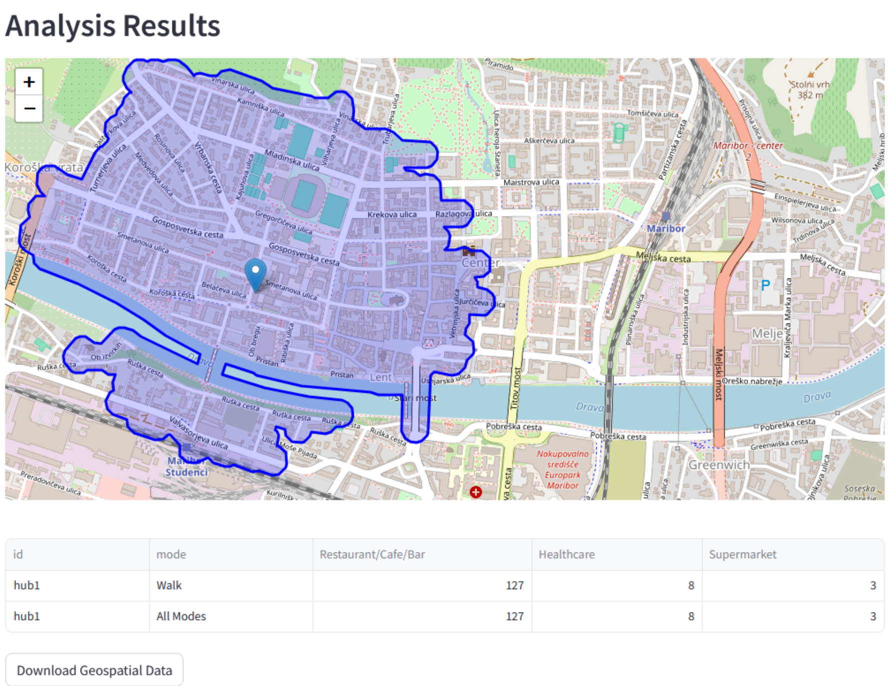

Figure 6.5: Results of analysis of access to restaurants, cafes, bars, healthcare and supermarkets within 15 minutes walk from the Faculty of Electrical Engineering and Computer Science, University of Maribor.

6.1.4 Accessibility measures

In [7] and [14], the authors introduce indices to measure various aspects of accessibility. To measure accessibility, some accessibility indicators are used. To introduce the x–minute city indicator, the following conditions should be considered:

- The city is divided to several blocks.

- A list of e destinations (establishments or amenities) classified by k categories of basic needs is given.
- The accessibility of a block is estimated as the total number of establishments per category that can be reached from a residence location (block) within the previously defined walking time threshold.

For each residential block $i \in \{1, \cdots, N\}$, the accessibility indicator A_i counts the number of opportunities that can be reached within an x-minute walk from the block i :

$$A_i = \sum_{1}^{e} E_j \cdot T,$$

where E_j is the number of establishments around the block i and T is 1 if the walking travel time threshold is a maximum of 15 min walking; otherwise $T = 0$.

In [7], for each block i within category k the diversity indicator $D_{i,k}$ is defined which incorporates the preference weight w_e of each establishment e, obtained from a ranking survey by gender, age, occupation, and other factors.

$$D_{i,k} = exp\left(\sum_{e\in k} log(A_i \cdot w_e)\right) / \sum_{e\in k} w_e,$$

where A_i is the normalized accessibility indicator.

They also define the walkability indicator WI_i, which includes variables relative to the quality of pedestrian infrastructure, security, and traffic:

$$WI_i = \left(\sum_{s\in i} WI_{s,i} \cdot L_{s,i}\right) / \sum_{s\in i} L_{s,i}$$

where $WI_{s,i}$ is the walkability value of the street segment s inside the isochrone of block i and $L_{s,i}$ is the length of each segment s in the 15-minute isochrone area of block i.

Also a 15-minute city index, P_{15i}, is defined as

$$P_{15i} = \sum_{k} D_{i,k} \cdot WI_i \quad \text{is the 15-m index for the block } i.$$

In [14], the authors develop and present a methodology to assess which cities are prepared to function as x-minute cities. The methodology was tested on the city of Sevilla, with a population of about 687,000 and an area of 114 km^2. The authors considered a population percentage indicator, which measures the proportion of residents with access to a given service. This indicator helps incorporate population density and connect spatial accessibility with the actual distribution of people.

For a block i and a category k, if the cumulative accessibility value for a given location (A_i) is greater than zero, the population living in that location has access to the given service. If the accessibility value is equal to zero at a given location, that means that the population at that location does not have access to the service.

$$AP = \left(\sum_{i} P_i \cdot g(A_i)\right) / \sum_{i} P_i$$

where P_i is the number of inhabitants living at block i; and the function $g(A_i)$ takes the value 0 if the cumulative index is negative and takes value 1 otherwise.

Also the index with extra variables is considered in the formulae of the indicators:

- The active modes $m \in$ {cycling , walking} (also public transport in large cities),

- The time thresholds $t \in \{5, 10, 15, ..\}$.

This allows them to compare the index depending on these new variants.

They concluded that in Sevilla most public services are accessible within a 10-minute walk, depending on the category. For the functional urban area, they found that public transport stops and bike stations are reachable by cycling.

6.2 Conclusions

This chapter has explored the concept of the 15-minute city that envisions essential services such as healthcare, education, work, food, leisure, and culture being accessible within a 15-minute walk or bike ride. Approached through the lens of combinatorial optimization and computational modeling, the review underscores that the 15-minute city is both a guiding urban planning framework and a complex computational challenge. Graph theory, optimization techniques, interactive tools, and accessibility indices emerge as complementary instruments for translating this vision into practice. The chapter emphasizes that rigorous mathematical modeling is crucial for shaping urban environments that are equitable, sustainable, and locally responsive.

Bibliography VI

[1] Taras Agryzkov, José L. Oliver, Leandro Tortosa, and José-Francisco Vicent. "Different types of graphs to model a city". In: *Computational Methods and Experimental Measurements* 118 (2017), pp. 71–82. doi: 10.2495/CMEM170081.

[2] Okan Arslan and Gilbert Laporte. "The 15-minute city concept: An operations research perspective and a research agenda". In: *Transportation Research Part E: Logistics and Transportation Review* 202 (2025), paper no. 104287. doi: 10.1016/j.tre.2025.104287.

[3] Lorenzo Barbieri, Roberto D'Autilia, Paola Marrone, and Ilaria Montella. "Graph representation of the 15-Minute City: A Comparison between Rome, London, and Paris". In: *Sustainability* 15.4 (2023), paper no. 3772. doi: 10.3390/su15043772.

[4] Matteo Bruno, Hygor Piaget Monteiro Melo, Bruno Campanelli, and Vittorio Loreto. "A universal framework for inclusive 15-minute cities". In: *Nature Cities* 1 (2024), pp. 633–641. doi: 10.1038/s44284-024-00119-4.

[5] Barbara Caselli, Silvia Rossetti Martina Carra, and Michele Zazzi. "Exploring the 15-minute neighbourhoods. An evaluation based on the walkability performance to public facilities". In: *Transportation Research Procedia* 60 (2022), pp. 346–353. doi: 10.1016/j.trpro.2021.12.045.

[6] Jose M. Gutierrez, Michael Jensen, and Tahir Riaz. "Applied graph theory to real smart city logistic problems". In: *Procedia Computer Science* 95 (2016), pp. 40–47. doi: 10.1016/j.procs.2016.09.291.

[7] Luis A. Guzman, Daniel Oviedo, and Victor A. Cantillo-Garcia. "Is proximity enough? A critical analysis of a 15-minute city considering individual perceptions". In: *Cities* 148 (2024), paper no. 104882. doi: 10.1016/j.cities.2024.104882.

[8] Weimin Huang and Elias B. Khalil. "Walkability optimization: formulations, algorithms, and a case study of Toronto". In: *Proceedings of the AAAI Conference on Artificial Intelligence* 37.12 (2023), pp. 14249–14258. doi: 10.1609/aaai.v37i12.26667.

[9] İstanbul Büyükşehir Belediyesi. *34 Dakika İstanbul*. İBB Akıllı Şehir Şube Müdürlüğü, 2025. url: https://34dakika.istanbul/ (visited on 09/18/2025).

[10] Hou Cheng Lam. "An Algorithmic Approach to the 15-minute City". Master's Thesis in Computer Science. University of Padova, 2023. url: https://thesis.unipd.it/handle/20.500.12608/68874 (visited on 09/19/2025).

[11] Carlos Moreno, Zaheer Allam, Didier Chabaud, Catherine Gall, and Florent Pratlong. "Introducing the "15-minute city": sustainability, resilience and place identity in future post-pandemic cities". In: *Smart Cities* 4.1 (2021), pp. 93–111. doi: 10.3390/smartcities4010006.

[12] Beatrice Olivari, Piergiorgio Cipriano, Maurizio Napolitano, and Luca Giovannini. "Are Italian cities already 15-minute? Presenting the next proximity index: a novel and scalable way to measure it, based on open data". In: *Journal of Urban Mobility* 4 (2023), paper no. 100057. doi: 10.1016/j.urbmob.2023.100057.

[13] Efthymis Papadopoulos, Alexandros Sdoukopoulos, and Ioannis Politis. "Measuring compliance with the 15-minute city concept: state-of-the-art, major components and further requirements". In: *Sustainable Cities and Society* 99 (2023), paper no. 104875. doi: 10.1016/j.scs.2023.104875.

[14] Miklós Radics, Panayotis Christidis, Borja Alonso, and Luigi dell'Olio. "The X-minute city: analysing accessibility to essential daily destinations by active mobility in Seville". In: *Land* 13.10 (2024), paper no. 1656. doi: 10.3390/land13101656.

[15] Debajit Sensarma. "Applications of graphs in smart cities". In: *Handbook of Research on Data-Driven Mathematical Modeling in Smart Cities*. IGI Global, 2023. Chap. 3. doi: 10.4018/978-1-6684-6408-3.ch003.

[16] SmartHubs Project. *Smart Mobility Hubs*. SmartHubs Consortium, 2025. url: https://www.smartmobilityhubs.eu/ (visited on 09/18/2025).

[17] Jianying Wang, Mei-Po Kwan, Gezhi Xiu, and Fangxu Deng. "A robust method for evaluating the potentials of 15-minute cities: implications for sustainable urban futures". In: *Geography and Sustainability* 5.4 (2024), pp. 597–606. doi: 10.1016/j.geosus.2024.07.004.

[18] Tianyi Wang, Yan Li, I-Ting Chuang, Weijie Qiao, Jing Jiang, and Lee Beattie. "Evaluating the 15-minute city paradigm across urban districts: a mobility-based approach in Hamilton, New Zealand". In: *Cities* 151 (2024), paper no. 105147. doi: 10.1016/j.cities.2024.105147.

Chapter VII

Urban Street Cleaning

This chapter studies urban street cleaning as a practical application of the arc routing problem. In contrast to traditional node routing formulations, the goal of arc routing is to service a subset of the network edges with a minimal cost, corresponding to the streets to be cleaned. An overview is given focusing on special constraints connected to this use-case such as capacities or time windows, and related application areas such as snow removal, waste collection or digital mapping are also explored.

Keywords: Capacitated arc routing, street cleaning, time windows, electric vehicles

VII

Urban Street Cleaning

Taner Bilgiç, Boğaziçi University, Türkiye
Balázs Dávid, University of Primorska, Slovenia
Gavin Le Roux, Stellenbosch University, South Africa
Nour Elhouda Tellache, University of Fribourg, Switzerland

Routing problems have been prevalent in operations research and combinatorial optimization for decades, with application areas such as transportation, logistics or telecommunications. At their core, the goal of these problems is to determine efficient routes for one or more agents traversing a network while minimizing travel distance or other arising costs. Classical examples include the Traveling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP), both of which are node-based formulations, where the routes are constructed through visiting the nodes of a network in an efficient order. Despite their simple concept, routing problems are usually computationally challenging, often NP-hard.

The street cleaning problem addressed in this work can be seen as a variant of the arc routing problem (ARP). Unlike node routing problems, which focus on visiting specific nodes while considering arcs primarily as connections, ARP emphasizes the traversal of arcs themselves. Beyond street cleaning, ARP arises in several practical contexts, including snow removal [17], salt spreading [21], road inspection [25], postal delivery, waste collection [3], and meter reading [10]. In applications such as meter reading, waste collection, and postal delivery, the density of demand along a street is sufficiently high that the entire segment can be treated as the relevant service unit.

7.1 Capacitated arc routing

Most of the above applications consider at least one resource type that is accumulated - or diminished - over the routes, which positions them as capacitated arc routing problems (CARP). This variation of ARP involves capacitated vehicles and assigns demands to certain edges. The total demand on a route cannot exceed the capacity of its vehicle. The following formal definition of CARP will closely follow the one given in [2].

Let $G = (V, E)$ be an undirected graph with edge costs $c_{ij} = c_{ji}$. Each edge $e(i, j) \in E$ is allocated a demand $q_{ij} \geq 0$. Edges with positive demand are called required arcs and $R = \{ e(i, j) \mid q_{ij} > 0 \}$, and have to be serviced by exactly one vehicle. The available fleet consists of K identical vehicles of capacity Q, assuming that $Q \geq q_{ij}$ for all (i, j). The nodes of the network are denoted by $\{1, \ldots, n\}$, with node 1 serving as the depot. Let $N(i)$ denote the nodes adjacent to i in G.

Based on the above, the integer programming formulation of CARP — as given in [16] — is the following:

- $x_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ traverses } e(i, j) \text{ from } i \text{ to } j, \\ 0 & \text{otherwise;} \end{cases}$
- $l_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ services } e(i, j) \text{ while traversing,} \\ 0 & \text{otherwise.} \end{cases}$

$$
\begin{aligned}
\min \; & \sum_{k=1}^{K} \sum_{(i,j)\in E} c_{ij}\, x_{ij}^{k} && \\
\text{s.t.} \; & \sum_{j \in N(i)} \left(x_{ij}^{k} - x_{ji}^{k}\right) = 0 && \forall\, i \in V,\; k = 1, \dots, K \\
& x_{ij}^{k} \;\geq\; l_{ij}^{k} && \forall\, (i,j) \in E,\; \forall\, k \\
& \sum_{k=1}^{K} \left(l_{ij}^{k} + l_{ji}^{k}\right) = 1 && \text{if } e(i,j) \in R \\
& \sum_{i} \sum_{j} l_{ij}^{k}\, q_{ij} \;\leq\; Q && \forall\, k \\
& \left.\begin{aligned}
& \sum_{(i,j)\in(S,S)} x_{ij}^{k} - n^2 y_S^k \leq |S| - 1 \\
& \sum_{(i,j)\in(S,\bar{S})} x_{ij}^{k} + u_S^k \geq 1 \\
& u_S^k + y_S^k \leq 1, \quad u_S^k, y_S^k \in \{0,1\}
\end{aligned}\right\} && \forall\, \emptyset \neq S \subseteq \{2, \dots, n\},\; \forall\, k \\
& x_{ij}^{k}, l_{ij}^{k} \in \{0,1\} && \forall\, (i,j) \in E,\; \forall\, S,\; \forall\, k
\end{aligned}
$$

Constraint (7.1) ensures route continuity for each vehicle. Constraint (7.1) states that serviced edges must be traversed. Constraint (7.1) forces all required arcs to be serviced. The explicit vehicle capacity is enforced by Constraint (7.1). The grouped constraints in (7.1) eliminate disconnected subtours but still allow solutions with two or more closed cycles. Naturally, the exponential number of constraints in (7.1) makes their direct use impractical, and models usually employ approaches such as the Miller-Tucker-Zemlin (MTZ) formulation instead to resolve this issue. Finally, Constraint (7.1) enforces the binary restrictions.

While there are other ARP variants that have efficient polynomial algorithms, most problems are hard to solve. Table 7.1 presents an overview of some of the most well-known ARP variants. For each of them, the edge set of the network (E — undirected edges, A — arcs) and the definition of the required edge set R is given, as well as the complexity of the problem variant (either in P, NP-complete, or NP-hard). It can be seen that the complexity of CARP is NP-hard [16].

Multiple methods have been proposed for transforming a CARP into a capacitated VRP (CVRP) by replacing required edges of the network with nodes to be serviced [14, 20, 23]. While these might prove to be efficient approaches for general graphs, all transformation techniques rely on the property that the triangle inequality holds in the original graph and optimal CARP tours will always perform deadheading along shortest paths [4]. However, this cannot be guaranteed in most real-world applications, such as the street cleaning problem.

ARP variant	**Edge set**	R	**Complexity**
Undirected postman	E	$R = E$	P
Directed postman	A	$R = A$	P
Mixed postman	A, E	$R = A \cup E,\ A \neq \emptyset,\ E \neq \emptyset$	NP-C
Rural postman	E	$R \subset E$	NP-C
Directed rural postman	A	$R \subset A$	NP-C
Capacitated arc routing	E	$R \subset E$	NP-H
Capacitated postman	E	$R = E$	NP-H

Table 7.1: Complexity of different arc routing variants.

7.2

The Street Cleaning Problem

The street cleaning problem, or similar variants of CARP have been studied for decades. An overview of the most important publications is presented in Table 7.2.

Author (Year)	**Edges**	**Time windows**	**Solution method**	**Application area**
Bodin and Kursh (1978) [6]	directed	hard	Heuristic	street sweeping
Eglese and Murdock (1991) [13]	directed	none	Heuristic	road sweeping
Eglese (1994) [12]	undirected	hard	Simulated annealing	winter gritting
Tagmouti et al. (2007) [26]	directed	hard	Exact	winter gritting
Johnson and Wøhlk (2009) [19]	undirected	hard	Exact, heuristc	none
Vansteenwegen et al. (2010) [28]	mixed	soft	Iterated local search	digital mapping vans
Afsar (2010) [1]	undirected	soft	Exact	none
Blazquez et al. (2012) [5]	directed	none	Exact, metaheuristic(NNS)	street sweeping
Zhang et al. (2018) [32]	directed	none	Metaheuristic (ACO+ALNS)	none
Yurtseven and Gokce (2019) [31]	directed	hard	Exact	street sweeping
Holmberg (2019) [17]	mixed	none	Hybrid	snow remover
Tirkolaee et al. (2020) [27]	undirected	none	Hybrid ACO	waste management
Jin et al. (2021) [18]	undirected	soft	Exact, NS	garbage collection
Chen et al. (2025) [7]	directed	none	Heuristic	road cleanning
Parsons et al. (2025) [22]	directed	none	Heuristic	street sweeping

Table 7.2: Comparative summary of the relevant literature ARP in street cleaning.

Street sweeping was first introduced as an application of arc routing in [6], where the authors incorporated strict time windows — referred to as the 'parking regulation constraint' — specifying when each street could be cleaned. In [12], Eglese addressed the routing of winter gritting vehicles, incorporating time windows that specify the maximum allowable time from the start of a shift to the service of a given street segment. The author proposed a simulated annealing approach to solve this problem.In their dissertation, Wøhlk [29] covers various aspects of CARP, presenting mathematical formulations and solution approaches to them. Building on this work, Johnson and Wøhlk [19] introduced a column generation method to address the CARP with time windows (CARP-TW).

Vansteenwegen [28] study CARP with soft time windows in the context of a mobile mapping van application for Tele Atlas. Their underlying arc routing problem is transformed into a vehicle routing problem. Service times are associated with preferred intervals, but violations are allowed at the expense of penalty costs.

The authors propose an iterated local search heuristic to solve the problem, without developing a mathematical model.

The paper [26] is also motivated by winter gritting, and studies a variant of CARP with time-dependent arc service costs. The problem is formulated and solved using a column generation framework.

Asfar [1] introduced the Capacitated Arc Routing Problem with Flexible Time Windows, where service is permitted outside the preferred time window at the expense of an additional penalty. This penalty is modeled as a piecewise linear function of earliness or lateness. To address the problem, the authors proposed a branch-and-price algorithm based on Dantzig–Wolfe decomposition.

Jin et al. [18] study arc routing problem for urban garbage collection, where servicing an edge within certain intervals incurs penalty costs. The problem is solved using a combination of heuristic methods and dynamic programming techniques.

Chen et al. [7] addressed the problem of planning operating routes for road cleaning fleets with the objective of optimizing both costs and drivers' route preferences. The latter are considered from two perspectives: visual attractiveness, which reflects drivers' preferences for certain routes, and operational attractiveness, which reflects preferences for the internal route structure. In their model, cleaning vehicles depart from a depot, replenish water from a water replenishment station, perform cleaning tasks, and visit intermediate facilities (water replenishment and sewage disposal stations) as needed. Upon completing their tasks, the vehicles dispose of sewage at a sewage station before returning to the depot, thereby completing the operation. To address this problem, the authors proposed heuristic methods based on Randomized Merge procedure [8], enhanced by two strategies designed to improve driver acceptance: imitating drivers' routing behavior to strengthen operational attractiveness, and reducing route overlap to enhance visual attractiveness.

Parsons et al. [22] investigated the street-sweeping problem in the city of Oshawa, Canada. The study distinguished between two road classes—arterial and collector (AC) roads and residential (RES) roads—analyzed separately due to differences in their configuration. Street-sweeping operations are conducted in Spring, Summer, and Fall, with each season presenting distinct conditions in terms of debris accumulation and operational principles. Each road segment must be swept as many times as there are curbs: one-way roads require two passes in the same direction of travel, while two-way roads require one pass in each direction. The road network also includes a central depot and temporary debris storage facilities, and the model accounts for several practical constraints, including shift length, fuel capacity, debris capacity, and water capacity. For the AC and RES classes, the authors first applied a two-stage clustering method, combining the weighted k-means algorithm with differential evolution, to divide the road network into operational areas. Within each area, they developed a routing algorithm to minimize the number of routes required to service all edges. This phase employed a three-step augment–merge algorithm to generate initial solutions. U-turns were subsequently reduced using a modified version of Hierholzer's algorithm combined with tabu search, and any remaining U-turns were eliminated through a forward-searching ant colony optimization procedure.

7.2.1 Other related routing problems

While not strictly dealing with street cleaning, this section will discuss research into other routing problem classes. These can also prove useful, as their constraints, objectives and even modeling techniques can share similar characteristics to the CARP formulations discussed above.

7.2.1.1 Route optimization

Zhang et al. [32] addressed the increasing use of electric vehicles (EVs) in distribution services due to environmental concerns over greenhouse gas emissions. Unlike conventional vehicles, EVs require en-route recharging due to limited battery capacity, making traditional routing schemes unsuitable. The study introduced the electric vehicle routing problem (VRP) (which may be considered a type of pollution-routing problem which extends on the VRP by considering fuel consumption costs, operational costs and greenhouse gas emissions), formulated a mathematical model aimed at minimizing energy consumption and incorporated a detailed energy usage calculation. To solve the problem, an ant colony (AC) algorithm and an adaptive large neighborhood search (ALNS) were tested on newly generated instances after which the former algorithm was proposed.

Their AC algorithm is hybridized with an iterated local search (ILS), marking the first application of this approach to solving the considered problem. The algorithm begins by generating traveling salesman problem (TSP) solutions based on a trail intensity matrix and a saving value to determine customer visit sequences. These solutions are converted to VRP and then electric VRP solutions by inserting depots and recharging stations according to energy and capacity constraints. The initial electric VRP solutions are improved using an ILS scheme, which perturbs and refines routes using four local search operators, namely 2-opt, relocate, exchange and station insertion/removal. To benchmark the AC algorithm, the authors adapted the ALNS by Goeke and Schneider [15] for the electric VRP. The ALNS begins with a randomly generated initial solution and iteratively applies destroy and repair operators, elected based on probabilities, to remove and reinsert customers. The solution is then improved using local search and accepted based on simulated annealing (SA) criteria.

A core issue in waste management systems is organizing operations in a way that maximizes efficiency and profit while minimizing financial costs. A major contributor to operational expenses in waste collection is the time spent by drivers, the usage time of collection vehicles and the total distance traveled. Wojciechowski et al. [30] presented a practical route optimization method for vehicles collecting both mixed and segregated urban waste. The approach focuses on determining the most efficient order for visiting city streets. The aim is to reduce unnecessary travels, such as empty runs and repeated passes over the same segments, which in turn shortens both distance and working time. Their proposed method offers a universal, adaptable framework making it applicable to improving vehicle routing in other areas and offering a starting point for further refinement and optimization of waste collection logistics.

The authors presented three heuristic/metaheuristic approaches, namely a nearest neighbor search, ant colony optimization (ACO) and a genetic algorithm (GA), to solve the considered problem, each with unique design choices tailored to the problem. Their nearest neighbor search limits route construction to a series of greedy choices based on local proximity, selecting the next closest unvisited street at each step and using a linear programming formulation to enforce that each location is visited exactly once. In the ACO algorithm, the authors specifically model pheromone deposition as inversely proportional to route length and incorporate pheromone evaporation to discourage convergence on longer routes. For the GA, the authors used TSP methodologies to select initial populations based on proximity to a starting point, applied problem-specific crossover techniques (including partially mapped, order and cycle crossovers) and implemented a diverse set of mutation operators (including inversion, insertion, relocation and mutual exchange) to introduce variability.

7.2.1.2 Arc routing problems for waste management

The literature on waste collection is typically categorized by waste type (commercial, residential and roll-on-roll-off) each with distinct operational characteristics, such as customer types, vehicle capacities and container sizes. Most studies aim to optimize vehicle routing to minimize distance and cost. Street sweeping, a key municipal waste management task, involves vehicles with limited bin capacity traversing both sides of streets and unloading at disposal sites, while also adhering to municipal restrictions like parking hours and noise regulations. The study by Yurtseven an Gökçe [31] introduces a novel perspective by incorporating electric-powered sweepers, which are quieter and more energy-efficient but introduce new challenges related to battery recharging.

The authors solved the electric street sweeping routing problem as an arc routing problem that aims to minimize the total energy consumption and operational costs of electric-powered sweepers while adhering to real-world constraints. The problem involves routing a heterogeneous fleet of electric sweepers, each with limited battery life, bin capacity and a water tank, across a network of city streets that vary in width, slope and length. Sweepers begin their routes fully charged from a central depot and must return after servicing all required arcs, some of which have time windows or require more frequent cleaning. The sweepers must also decide when and where to recharge their batteries and empty their waste at one of the many disposal sites. Additional constraints include driver rest and lunch breaks. The authors proposed a mixed integer programming model and solved it using the CPLEX 12.8 solver. Lastly, they presented a case study consisting of a small street area in Izmir, Turkey.

On the other hand, the study by Babaee Tirkolaee et al. [27] addressed the urban solid waste management problem by formulating a multi-trip capacitated arc routing problem that accounts for the uncertain nature of waste generation using chance-constrained programming. The formulation is based on fuzzy credibility theory with the objective of minimizing total operational costs. The problem was modeled on an undirected graph where the depot and disposal facilities are located at separate nodes and each required edge, representing streets needing service, is served by exactly one vehicle which may undertake multiple trips. To solve this complex problem efficiently, the authors proposed a hybrid augmented ACO algorithm, similar to that by Wojciechowski et al. [30], integrating an improved max-min ant system with a novel probability function and SA. The Taguchi method was used to perform parameter tuning. The model incorporates key real-world constraints such as heterogeneous vehicles, fuzzy edge demand and an urban-to-outskirts routing structure.

The proposed metaheuristic is distinguished by its strategic integration of demand-awareness and sophisticated pheromone management. Unlike standard ACO procedures, the authors introduced a demand-sensitive probability function that biases ants towards high-demand edges earlier in the solution process, directly fitting the street-sweeping context. Additionally, their version of the max-min ant system used a pheromone trail smoothing mechanism, which adaptively adjusts pheromone levels based on deviation from upper bounds. This mitigates the risk of early convergence to near-optimal solutions. They further enhanced this by applying SA to refine a pool of 200 initial solutions generated by the greedy randomized adaptive search procedure, ensuring the pheromone matrix begins from a strong foundation. Their customized SA implementation includes five specialized local search operators, such as intra- and inter-route edge swaps and classical 2-opt and 3-opt moves, designed to account for electric vehicle constraints like battery limits and service time windows.

7.2.1.3 **Chinese postman problems**

The Chinese postman problem (CPP) is a classic problem in graph theory that asks for the shortest closed path or circuit that visits every edge of a graph at least once. The arc routing problem is a generalization of the CPP. While the Chinese Postman Problem (CPP) requires all edges in the graph to be serviced, the Arc Routing Problem (ARP) considers only a subset of edges, referred to as required arcs, that must be visited. These arcs may include streets that require cleaning, garbage collection or mail delivery. It is therefore worth investigating the literature on CPPs. Two papers have been identified in the literature survey by Corberán et al. [9].

Firstly, Corberán et al. [11] considered a maximum benefit Chinese postman problem (MBCPP), which is an NP-hard extension of the classic CPP, where each edge yields multiple benefits depending on how many times it is serviced. The goal is to determine a closed walk that maximizes the total accumulated benefit. To tackle this, the authors presented an integer programming formulation for the undirected version of the problem and developed a branch-and-cut (B&C) algorithm based on the polyhedral structure of the solution space. The B&C algorithm improves on the linear programming (LP) relaxation by iteratively adding violated inequalities through a cutting-plane approach. Initially, the LP includes key constraints, such as parity and variable bounds. At each iteration, violated inequalities are identified using separation algorithms. These include heuristic and exact methods for detecting connectivity, parity K-C and p-connectivity violations based on support graphs derived from fractional LP solutions. The algorithm prioritizes fast heuristics to limit computational effort, only applying exact procedures when necessary. Their method was tested computationally on instances with up to 1,000 vertices and 3,000 edges, demonstrating its scalability and effectiveness.

Similarly, the research proposed by Shafahi and Haghani [24] introduced the generalized maximum benefit k-Chinese postman problem, which extends on both the maximum benefit and multiple vehicle CPPs. The authors developed a novel mixed integer programming formulation capable of handling multiple realistic routing scenarios. Four distinct cases of the problem are considered to reflect varying operational constraints. In the first case, arc-routing is performed with fixed profits and each vehicle is required to start and end its route at the same user-defined location. The second case relaxes this by allowing the user to select different possible origins and destinations for each vehicle. The third case keeps the origin fixed but lets the model determine the most beneficial destination. The fourth and most flexible case allows the model to freely choose both origins and destinations for each vehicle, optimizing overall benefit. The model is applied to a real-world security patrolling problem on the University of Maryland campus network. The authors did not report on any solution methods used to solve the four cases.

7.3 Conclusions

This chapter reviewed the literature on urban street cleaning by framing it as a variant of the capacitated arc routing problem (CARP), where streets requiring service must be scheduled under constraints such as vehicle capacity and time windows. After outlining mathematical formulations of CARP, its variants, and their computational complexity in relation to vehicle and node routing models, the discussion turned to applications beyond street cleaning, including winter gritting, waste collection, and digital mapping. Within this context, solution approaches were shown to evolve from early heuristics and neighborhood search techniques to exact algorithms, branch-and-price methods, and advanced metaheuristics, with foundational insights drawn from classical problems such as the Chinese Postman Problem.

This review highlights that the literature on urban street cleaning and related real-world applications of CARP remains limited. Most proposed methods target specific use cases, often reformulating the underlying CARP as a node-routing variant. Furthermore, the development of generally efficient approaches that can jointly address multiple practical constraints, such as vehicle capacities, time windows, U-turn restrictions, and longer planning horizons, has yet to be fully explored.

Bibliography VII

[1] H. Murat Afsar. "A branch-and-price algorithm for capacitated arc routing problem with flexible time windows". In: *Electronic Notes in Discrete Mathematics* 36 (2010), pp. 319–326. doi: 10.1016/j.endm.2010.05.041.

[2] Arjang A. Assad. "Arc routing methods and applications". In: *Network Routing*. Vol. 8. Handbooks in Operations Research and Management Science. Elsevier, 1995, pp. 375–483. doi: 10.1016/S0927-0507(05)80109-4.

[3] Erfan Babaee Tirkolaee, Alireza Goli, Maryam Pahlevan, and Ramina Malekalipour Kordestanizadeh. "A robust bi-objective multi-trip periodic capacitated arc routing problem for urban waste collection using a multi-objective invasive weed optimization". In: *Waste Management & Research* 37.11 (2019), pp. 1089–1101. doi: 10.1177/0734242X19865340.

[4] José Manuel Belenguer, Enrique Benavent, and Stefan Irnich. "Chapter 9: The Capacitated Arc Routing Problem: Exact Algorithms". In: *Arc Routing*. SIAM, 2015, pp. 183–221. doi: 10.1137/1.9781611973679.ch9.

[5] Carola A. Blazquez, Alejandra Beghelli, and Veronica P. Meneses. "A novel methodology for determining low-cost fine particulate matter street sweeping routes". In: *Journal of the Air & Waste Management Association* 62.2 (2012), pp. 242–251. doi: 10.1080/10473289.2011.646048.

[6] Lawrence D. Bodin and Samuel J. Kursh. "A computer-assisted system for the routing and scheduling of street sweepers". In: *Operations Research* 26.4 (1978), pp. 525–537. doi: 10.1287/opre.26.4.525.

[7] Mingwei Chen, Xiaofang Wang, Linning Cai, and Liang Ma. "Designing visually and operationally attractive routes to improve driver acceptance in road cleaning vehicle routing problem". In: *Computers & Operations Research* 177 (2025), paper no. 106973. doi: 10.1016/j.cor.2025.106973.

[8] Geoff Clarke and John W. Wright. "Scheduling of vehicles from a central depot to a number of delivery points". In: *Operations Research* 12.4 (1964), pp. 568–581. url: https://www.jstor.org/stable/167703.

[9] Ángel Corberán, Richard Eglese, Geir Hasle, Isaac Plana, and José M. Sanchis. "Arc routing problems: A review of the past, present, and future". In: *Networks* 77.1 (2021), pp. 88–115. doi: 10.1002/net.21965.

[10] Ángel Corberán, Isaac Plana, Miguel Reula, and José M Sanchis. "On the distance-constrained close enough arc routing problem". In: *European Journal of Operational Research* 291.1 (2021), pp. 32–51. doi: 10.1016/j.ejor.2020.09.012.

[11] Ángel Corberán, Isaac Plana, Antonio M. Rodrìguez-Chìa, and José M. Sanchis. "A branch-and-cut algorithm for the maximum benefit Chinese postman problem". In: *Mathematical Programming* 141.1 (2013), pp. 21–48. doi: 10.1007/s10107-011-0507-6.

[12] Richard W. Eglese. "Routeing winter gritting vehicles". In: *Discrete Applied Mathematics* 48.3 (1994), pp. 231–244. doi: 10.1016/0166-218X(92)00003-5.

[13] Richard W. Eglese and Heather Murdock. "Routeing road sweepers in a rural area". In: *Journal of the Operational Research Society* 42.4 (1991), pp. 281–288. URL: https://www.jstor.org/stable/2583381.

[14] Les Foulds, Humberto Longo, and Jean Martins. "A compact transformation of arc routing problems into node routing problems". In: *Annals of Operations Research* 226.1 (2015), pp. 177–200. DOI: 10.1007/s10479-014-1732-1.

[15] Dominik Goeke and Michael Schneider. "Routing a mixed fleet of electric and conventional vehicles". In: *European Journal of Operational Research* 245.1 (2015), pp. 81–99. DOI: 10.1016/j.ejor.2015.01.049.

[16] Bruce L. Golden and Richard T. Wong. "Capacitated arc routing problems". In: *Networks* 11.3 (1981), pp. 305–315. DOI: 10.1002/net.3230110308.

[17] Kaj Holmberg. "The (over) zealous snow remover problem". In: *Transportation Science* 53.3 (2019), pp. 867–881. DOI: 10.1287/trsc.2018.0851.

[18] Xin Jin, Hu Qin, Zizhen Zhang, Mengchu Zhou, and Jiahai Wang. "Planning of Garbage Collection Service: An Arc-Routing Problem With Time-Dependent Penalty Cost". In: *IEEE Transactions on Intelligent Transportation Systems* 22.5 (2021), pp. 2692–2705. DOI: 10.1109/TITS.2020.2973806.

[19] Ellis L. Johnson and Sanne Wøhlk. *Solving the Capacitated Arc Routing Problem with Time Windows using Column Generation*. CORAL Working Paper L-2008-09. Aarhus School of Business, Aarhus University, Department of Business Studies, 2009. URL: https://pure.au.dk/portal/en/publications/solving-the-capacitated-arc-routing-problem-with-time-windows-usi (visited on 09/26/2025).

[20] Humberto Longo, Marcus Poggi de Aragão, and Eduardo Uchoa. "Solving capacitated arc routing problems using a transformation to the CVRP". In: *Computers & Operations Research* 33.6 (2006), pp. 1823–1837. DOI: 10.1016/j.cor.2004.11.020.

[21] Luc Muyldermans, Dirk Cattrysse, Dirk Van Oudheusden, and Tsippy Lotan. "Districting for salt spreading operations". In: *European Journal of Operational Research* 139.3 (2002), pp. 521–532. DOI: 10.1016/S0377-2217(01)00184-9.

[22] Tyler Parsons, Jaho Seo, and Dan Livesey. "Municipal street-sweeping area generation with route optimization". In: *International Transactions in Operational Research* 32.3 (2025), pp. 1375–1399. DOI: 10.1111/itor.13515.

[23] Wen-Lea Pearn, Arjang Assad, and Bruce L. Golden. "Transforming arc routing into node routing problems". In: *Computers & Operations Research* 14.4 (1987), pp. 285–288. DOI: 10.1016/0305-0548(87)90065-7.

[24] Ali Shafahi and Ali Haghani. "Generalized maximum benefit multiple Chinese postman problem". In: *Transportation Research Part C: Emerging Technologies* 55.1 (2015), pp. 261–272. DOI: 10.1016/j.trc.2015.01.017.

[25] Nichalin S. Summerfield, Moshe Dror, and Morris A. Cohen. "City streets parking enforcement inspection decisions: The Chinese postman's perspective". In: *European Journal of Operational Research* 242.1 (2015), pp. 149–160. DOI: 10.1016/j.ejor.2014.10.039.

[26] Mariam Tagmouti, Michel Gendreau, and Jean-Yves Potvin. "Arc routing problems with time-dependent service costs". In: *European Journal of Operational Research* 181.1 (2007), pp. 30–39. doi: 10.1016/j.ejor.2006.06.028.

[27] Erfan Babaee Tirkolaee, Iraj Mahdavi, Mir Mehdi Seyyed Esfahani, and Gerhard-Wilhelm Weber. "A hybrid augmented ant colony optimization for the multi-trip capacitated arc routing problem under fuzzy demands for urban solid waste management". In: *Waste Management & Research* 38.2 (2020), pp. 156–172. doi: 10.1177/0734242X198657.

[28] Pieter Vansteenwegen, Wouter Souffriau, and Kenneth Sörensen. "Solving the mobile mapping van problem: A hybrid metaheuristic for capacitated arc routing with soft time windows". In: *Computers & Operations Research* 37.11 (2010), pp. 1870–1876. doi: 10.1016/j.cor.2009.05.006.

[29] Sanne Wøhlk. "Contributions to Arc Routing". PhD thesis. University of Southern Denmark, 2006.

[30] Łukasz Wojciechowski, Tadeusz Cisowski, and Arkadiusz Małek. "Route optimization for city cleaning vehicle". In: *Open Engineering* 11.1 (2021), pp. 483–498. doi: 10.1515/eng-2021-0049.

[31] Cansu Yurtseven and Mahmut Ali Gökçe. "A novel arc-routing problem of electric powered street sweepers with time windows and intermediate stops". In: *IFAC-PapersOnLine* 52.13 (2019), pp. 2308–2313. doi: 10.1016/j.ifacol.2019.11.550.

[32] Shuai Zhang, Yuvraj Gajpal, S.S. Appadoo, and M.M.S. Abdulkader. "Electric vehicle routing problem with recharging stations for minimizing energy consumption". In: *International Journal of Production Economics* 203.1 (2018), pp. 404–413. doi: 10.1016/j.ijpe.2018.07.016.

Chapter VIII

Mobile Clinics

This chapter is motivated by a practical application of vehicle routing problems (VRPs) to mobile clinic routing and scheduling in rural South Africa. The mobile clinics case requires designing efficient multi-depot, multi-vehicle routes while balancing staff workloads, minimizing travel, and ensuring continuity of patient care. The problem aligns closely with home health care routing, and periodic vehicle routing problems, and is shaped by both logistical and service-related constraints. This problem illustrates how combinatorial optimization techniques can support healthcare logistics, improving both cost-effectiveness and equity of service provision.

Keywords: Mobile clinics, vehicle routing problem, multi-objective optimization.

VIII

Mobile Clinics Routing and Scheduling

Elise Bangerter, University of Freiborg, Switzerland
Taner Bilgiç, Boğaziçi University, Türkiye
Carmen-Ana Dominquez-Bravo, Universidad de Cádiz, Spain
Linke Potgieter, Stellenbosch University, South Africa
Gavin Le Roux, Stellenbosch University, South Africa

Remote regions often have limited access to healthcare and long distances have to be covered to access nearest facilities. To address this, mobile clinics are often used to improve access to healthcare in remote regions. In this chapter, mathematical approaches in literature to improve location and routing efficiency for similar real world problems are presented.

The mobile clinic setting that inspired this chapter is in the Witzenberg region, located in the Western Cape in South Africa. It is a mountainous area comprised mostly of farmland (about 148 farms) and farm workers living on the farms ranging from 10 to over 1000. Farm workers typically do not have their own transport but rely on their employee (farmers) to transport them to nearest towns when necessary. Within the same region, there are only eight healthcare facilities and one district hospital. These factors make it difficult for the farm worker population to access care. Three mobile clinics have been deployed in the region, to make primary healthcare more accessible to the farming communities. Each mobile clinic has an assigned daily route that services pre-specified farms according to a monthly schedule. In order to reduce communication barriers and provide continuity of care on farms, the farm visits follow a predictable schedule that is easy to remember for farm workers, and the farms are visited by the same mobile clinic each month. The three mobile clinics are based at two different clinics in the region and are serviced by staff from these clinics. In cases where they cannot complete the work on the assigned farm on the day, they return the next day. The routes and schedules for the mobile clinics in the Witzenberg region were created years ago, and the efficiency and fairness of these routes between the mobile clinics have been under scrutiny. Routes overlap between mobile clinics, and the workload is not evenly spread between the mobile clinics, where some mobile clinics serve larger farming communities than others (and hence more patients), and some mobile clinics are covering longer distances but serving smaller communities.

The mobile clinic mathematical problem considered in this chapter involves assigning a set of mobile clinics to a set of farms over multiple days, ensuring that each mobile clinic's daily schedule fits within their working hours, and ensuring that all farms are visited at least once over a 4-week period. Each mobile clinic starts and returns to the same depot each day, but there are multiple depots from where mobile clinics are operated. The objectives are to minimize total distance over all mobile clinics and maximize an equitable distribution of working time and travel time between the mobile clinics.

In a study by Callaghan et al. [3], a three-stage heuristic model was presented to solve the multi-depot periodic vehicle routing problem with fairness for the Witzenberg region. This model follows a multi-vehicle routing problem approach in the first stage for determining all the daily routes for the three mobile clinics that minimizes transport cost, a knapsack problem for distributing daily routes fairly amongst the mobile clinics in the second stage, and a vehicle routing problem approach in the third stage to schedule the routes in such a way that transport cost would be minimized should they have to return to a farm from the previous day (unfinished work), before continuing with the new daily route. The model was solved using a branch-cut-and-price algorithm for the first and third phases, and a branch-and-bound method for the second phase.

A list of literature related to the mobile clinics problem is given in Table 8.1. Information on the type of problem, whether uncertainties were considered, different approaches, whether continuity of care was considered, and whether fairness was considered is given. Mobile clinic related literature are mostly focused on the location of facilities problem, whereas the problem as defined here is a periodic vehicle routing problem. When considering the home health care problem in literature, it is found to be a more similar problem to the mobile clinics problem considered in this study.

The home health care problem is therefore reviewed first, in §8.1. Heuristics that solve related problems are then looked at in §8.2. Metaheuristics, in particular neighborhood search algorithms and evolutionary metaheuristics, are summarized in §8.2.1 and §8.2.2, respectively. The literature review closes off in §8.3 with optimization approaches that use exact algorithms in their implementation. Vehicle routing problems, location routing problems, covering tour problems, location-routing problems, facility location problems and scheduling problems are found in this section, since the methods used to solve them may be useful to solve the one considered in this study.

8.1 Home health care problem

The mobile clinic routing and scheduling problem is related to the home health care routing and scheduling problem. Decerle et al. [4] looked at the home health care (HHC) problem which involves assigning a set of caregivers to patient visits within a single day, ensuring each caregiver's schedule fits within their working hours and starts and ends at a designated office. Visits must begin within specific time windows agreed upon with patients, with varying degrees of flexibility based on the care required. Only compatible caregivers can perform certain visits, and some visits require synchronized arrival by two caregivers. The objectives are to minimize total travel time, maximize service quality (especially timely and synchronized visits) and minimize the maximal working time difference between the caregivers. They formulated the problem as a mixed-integer linear programming model which may be considered as a multi-depot vehicle routing problem (VRP) with time windows, synchronization constraints and workload balance.

They proposed a hybrid metaheuristic combining ant colony optimization (ACO) with a memetic algorithm (MA) to solve the problem. Each solution in the population is represented as a table of caregiver routes, which may vary in length. This direct encoding eliminates the need for decoding during optimization. The algorithm maintains a shared population of feasible solutions and alternates between applying MA and ACO operators at each iteration, based on a fixed probability parameter. If the MA is selected, traditional genetic operators like selection, crossover and mutation are applied. Otherwise, new individuals are generated using the ACO procedure. Throughout the process, only feasible solutions are maintained. All operators are therefore designed to preserve feasibility without requiring any repair mechanisms.

Frifita and Masmoudi [7] extended the work by Decerle et al. [4] by considering a VRP with time windows, temporal dependencies, multi-structures and multispecialties. Temporal dependencies include synchronization (similar as in the paper by Decerle et al. where several caregivers need to arrive at a given patient), precedence and disjunction. Precedence constraints enforce a strict ordering ensuring that one visit must be completed before another can begin. Disjunction constraints, on the other hand, enable mutual exclusivity in the sense that only one specialty from a set of alternative specialties can be assigned to a patient during a visit. Regarding the multi-structures component of the problem, this component refers to the involvement of multiple home care organizations, each offering different types of services such as social assistance, nursing care and medical services.

The authors developed three variable neighborhood search (VNS) algorithms, namely a standard VNS,

Table 8.1: Summary of papers for the literature survey.

Year	Author(s)	Application	Problem type	Objectives	Uncer-tainties	Solution approach		Continui-ty of care	Fairness
						Type	Method		
1998	Hodgson et al. [11]	Mobile clinics	Covering tour problem	Single	No	Exact	Branch-and-cut algorithm	Yes	No
2007	Doerner et al. [5]	Mobile healthcare	Location routing	Multiple	No	Metaheuristic	Genetic algorithm and ACO	No	No
2016	Ares et al. [1]	Mobile clinics	Facility location	Multiple	No	Exact	Column generation	No	Yes
2019	Decerle et al. [4]	Home healthcare	Time windows, synchronization	Multiple	No	Metaheuristic	Hybrid memetic-ACO algorithm	No	Yes
2019	Shi et al. [16]	Home healthcare	HHC routing and scheduling problem, robust optimization	Multiple	Yes	Metaheuristic	Robust optimization applied to SA, TS and VNS	No	No
2020	Frifita and Masmoudi [7]	Home healthcare	Temporal dependencies, multiple structures, specialties	Single	No	Metaheuristic	VNS	No	No
2021	Fathollahi-Fard et al. [6]	Home healthcare	Working-time balancing, continuity of care	Multiple	Yes	Metaheuristic	Multi-objective red deer algorithm	Yes	Yes
2022	Savaşer and Kara [15]	Mobile clinics	Periodic location routing	Single	No	Matheuristic	Cluster first-route second	Yes	No
2022	Bayraktar et al. [2]	Mobile clinics	Multi-period mobile facility location problem with mobile demand	Single	No	Metaheuristic	ALNS	No	No
2023	Santa Gonzalez et al. [10]	Mobile clinics	Multi-period location routing problem	Single	No	Heuristic	Route generation algorithm	No	No
2024	Fu et al. [8]	Home healthcare	HHC routing and scheduling problem with stochastic optimization	Multiple	Yes	Metaheuristic	Change constrained and multi-objective migrating birds optimization	No	Yes
2024	Liu et al. [12]	Home healthcare	Multi-depot capacitated	Multiple	No	Metaheuristic	AMOALNS	No	Yes
2024	Zhao et al. [18]	Home healthcare	HHC routing and scheduling, problem under uncertainty	Multiple	Yes	Metaheuristic	Stochastic ALNS embedded with a local search	Yes	Yes
2025	Callaghan et al. [3]	Mobile clinics	Multi-depot periodic capacitated VRP with fairness	Multiple	No	Matheuristic	Route first-cluster second	Yes	Yes

a VNS hybridized with several local searches (GVNS) and GVNS with an ejection chains (EC) based local search. Six neighborhood structures are proposed. Three of the intra-route operators include exchanging the positions of two adjacent nodes in the same route (adjacent swap), replacing a pair of nonadjacent arcs with a new pair (2-opt) and removing a set of adjacent nodes followed by inserting them between other adjacent nodes in the route (block insertion). The three inter-route operators exchange the positions of any two nodes located in different routes (swap), move a sequence of visits from a larger route into a smaller one at a random position (shift) and randomly select a set of visits from each route and permute them (OR-OPT). Lastly, there are four local searches that are implemented in the VNS algorithm.

1. LS1 detects visits violating time windows and temporal dependencies and reinserts them into the best feasible position in the same or different route.
2. LS2 randomly selects a visit and reinserts it into the best feasible position, repeated 100 times.
3. LS3 applies LS1 followed by LS2.
4. The EC-based local search randomly ejects a visit from a route and inserts it into another. Further visits are possibly ejected from routes in which ejected visits were inserted creating a chain. This process is continued until the capacity is maintained after which the resulting feasible solution is kept in memory.

Fathollahi-Fard et al. [6] studied a similar problem to those by Decerle et al. [4] and Frifita & Masmoudi [7]. Their study, also thought of as an HHC problem, was motivated by the caring of elderly individuals and providing general hygiene assistance in patients' homes. A solid strategy for ensuring sustainable HHC operations involves planning that balances caregivers' working hours, maintains continuity of care and accounts for various uncertainties, such as patient availability, service and travel times as well as organizational regulations aimed at upholding high-quality care standards. As such, both deterministic and probabilistic formulations are provided. The problem is modeled as a multi-depot multiperiod multi-service VRP with time windows. The objectives are to minimize the total cost of the system, minimize the idle time of the caregivers and optimize the continuity of care (by minimizing the maximum number of patients for each caregiver as a min-max function).

Three heuristics were developed to solve the problem. The first heuristic begins each caregiver's tour with the patient closest to the pharmacy, then sequentially adds the next nearest patients based on travel cost. The second heuristic starts the tour with the patient who, on average, is closest to all others (based on the travel cost matrix), then adds the nearest patients in the sequence. The third heuristic initiates the tour with the patient furthest from the laboratory, then builds the rest of the tour by adding the nearest patients based on travel cost. Moreover, they developed two nature-inspired metaheuristics, namely a multi-objective red deer algorithm (MORDA) and an improved version of it. MORDA works by evolving a population of candidate solutions through stochastic events such as roaring, fighting and mating, aiming to approximate the Pareto front of efficient solutions. The improved version introduces adaptive strategies to fine-tune its exploration and exploitation balance.

A more recent study solving the HHC problem in a pandemic context is by Liu et al. [12]. In the routing and scheduling problem, unique pandemic constraints such as caregiver contact time limits and multi-trip routes are considered. Furthermore, the problem accounts for synchronized visits, lunch breaks and time windows. The goal is to assign caregivers to patients and optimize routes while minimizing travel costs, workload imbalance and patient dissatisfaction.

The authors proposed a hybrid metaheuristic that combines an archived multi-objective simulated annealing (AMOSA) algorithm with an adaptive large neighborhood search (ALNS). It constructs new solutions iteratively using problem-specific heuristics embedded within the ALNS framework, operating under a multi-objective optimization structure that strategically ruins and recreates feasible solutions. The heuristic weights are adaptively updated based on the dominance relationship between the newly generated solution and the existing non-dominated solutions stored in the AMOSA archive. Extensive experiments demonstrate its effectiveness with results highlighting that workload balance and patient preference can often be optimized together. However, minimizing travel costs remains the most challenging objective.

Home Health Care Routing and Scheduling Problems (HHCRSP) literature has increasingly moved beyond classical deterministic formulations to address uncertainty in travel and service times. Shi et al. [16] tackle this challenge by introducing a robust optimization framework that captures variability through a budget-uncertainty set rather than explicit probability distributions. Their mixed-integer linear model accounts for time-window constraints and the propagation of delays, and is solved via a hybrid strategy combining exact optimization and metaheuristics. To validate performance, the authors stress-test solutions through Monte Carlo simulation, demonstrating that the robust approach consistently produces reliable schedules with controlled sensitivity to uncertainty.

Building on this foundation, Fu et al. [8] propose a stochastic, multi-objective HHCRSP that explicitly incorporates uncertain travel and service times along with caregiver workload balance. Their formulation adopts a chance-constrained programming approach and is solved using a Multi-Objective Migrating Birds optimization algorithm enhanced with stochastic simulation to assess feasibility and solution quality.

In a similar vein, Zhao et al. [18] address a bi-objective HHCRSP under uncertainty, incorporating caregiver qualifications and workload into the modeling of variable service times. The authors develop a mixed-integer linear programming model and tackle its complexity using a stochastic Adaptive Large Neighborhood Search embedded within an Enhanced Multi-Directional Local Search (ALNS-EMDLS). This hybrid heuristic framework generates Pareto-optimal solutions that are tested for robustness through sensitivity analysis, highlighting the trade-offs between efficiency and service reliability in uncertain environments.

8.2 Heuristics

Savaşer and Kara [15] looked at the planning of mobile health care services in rural areas where doctors visit remote villages which do not have a health care facility nearby. They produce monthly schedules that contain assignments of doctors to villages, their visit schedules and where they start and end their tour. The objective is to increase coverage of such population centres. They modeled the problem as a period location routing problem subject to constraints imposed by regulations from the Ministry of Health of Turkey. These constraints include continuity of care and evenly distributed periodic visits. For example, there are deterministic rules on frequency of visits based on the population of the visited village. Demand is not explicitly modeled and a coverage approach is assumed to satisfy the demand.

They developed an integer programming formulation and propose a heuristic algorithm based on a cluster first-route second approach to solve realistic instances of the problem. Their algorithm works by exploiting the hierarchical relationship between the assignment and routing decisions. In the first stage, villages are allocated to doctors while ensuring that working hour constraints are not exceeded. In the second stage, the optimal route for each doctor is determined individually based on their assigned villages so that the total travel distance is minimized.

8.2.1 Neighborhood search algorithms

Three papers [2, 13, 17] during the period 2022–2024 are presented on neighborhood search algorithms. Bayraktar et al. [2] studied relief aid provision to en route refugees. Refugee groups can enter and leave the network at different time period, following predefined paths. During each period, a group moves from its current node to a neighboring one along this path. Facilities must be placed at specific nodes in each period to deliver service to the refugees located there. Additionally, every refugee group must receive service at least once within a specified number of consecutive time periods. The authors addressed it as a multi-period facility location problem in which both the facilities and demand are mobile on a network. The problem is to locate the facilities in each period to minimize the total setup and travel costs of the mobile facilities while ensuring the service requirement is met. The aim is to determine the locations, and thus the routes, of the mobile facilities during the planning horizon to satisfy the refugee service requirements while refugees follow their paths.

The authors developed an ALNS to solve the problem. They proposed three destroy operators and four repair operators. The three destroy operators are explained first. The first operator randomly selects and removes individual service acts provided by randomly chosen mobile facilities. The second operator removes the entire visit schedule and associated service acts of randomly chosen mobile facilities. The third operator uses a relatedness-based heuristic to remove service acts that are similar in terms of location, time period and refugee groups services. The first repair operator randomly selects a minimal feasible service act sequence (FSAS) for an uncovered path and inserts its missing service acts into the visit schedules of mobile facilities. The second operator chooses the minimal FSAS for an uncovered path tat causes the smallest increase in the objective function value and inserts its missing service acts into the solution. The third operator selects the most costly FSAS and inserts all its missing service acts. The last operator ranks all FSASs of each uncovered path by cost and insert the one with the largest regret value.

A key challenge in horizontal collaborative transportation is ensuring a fair distribution of benefits among partners, as traditional gain-sharing methods often leave some partners feeling unfairly treated. The study by Soriano et al. [17] introduces the multi-depot VRP with profit fairness, a bi-objective optimization problem that incorporates a fairness objective alongside the standard cost minimization objective, to address this problem. By analyzing the model, the paper investigates how integrating fairness impacts the optimization process. To approximate the Pareto front for each problem instance, an ALNS is used within an ϵ-constraint framework. The model is tested across various instance types and planning horizons and the resulting Pareto fronts are evaluated using different performance metrics.

The six destruction operators each remove z nodes from the current solution using different strategies. The random operator selects nodes entirely at random while the worst operator removes those whose removal yields the greatest decrease in travel distance. The related operator starts with one random node and iteratively remove the most closely related nodes based on distance and the route operator targets routes with high travel-per-customer ratios. The historical operator uses past performance data to identify and remove nodes frequently involves in poor solutions while the proximity operator removes nodes that are closest to depots other than their current assignment. Two reconstruction operators are defined using insertion scores. Greedy insertion selects nodes with the lowest insertion cost, choosing randomly among the top three, while regret-κ insertion prioritises nodes with the largest gap between their best and κ-th best insertion options.

The work by Londoño et al. [13] is similar to the research by Frifita & Masmoudi [7]. They looked at a multi-objective multi-depot VRP in which the distance traveled by the vehicles and the standard deviation of the routes across all vehicles are minimized. The second objective function addresses fair work balance by ensuring that the routes are of approximately equal length. The main contribution of the study is the hybridization of the Chu-Beasley genetic algorithm (GA) and VNS, where the latter is embedded within the former, a strategy that has shown strong performance in previous multi-depot VRPs. To support decision-making, the results are presented as a set of non-dominated solutions forming a Pareto front.

The Chu-Beasley GA is a modification of the original GA in the sense that a single individual is changed in each generation as opposed to the entire population. In their proposed algorithm, one-point crossover is applied to two randomly selected parents by tournament selection. After crossover, only one offspring is passed to the next stage together with the remaining parents. The offspring may result in infeasible solutions if depots are misplaced in the routes, which are penalized in the fitness function. After the mutation stage, VNS is performed as a local search on the resulting offspring. It utilizes three intra-operators, namely relocating a customer within the same route between adjacent customers, swapping two customers within the same route and reversing the sequence of consecutive customers within a single route. It also employs three inter-operators, namely relocating a customer from one route to another, swapping two customers from different routes as well as reversing a sequence of customers across two routes and inserting the resulting reversed sequence in the other route of the two routes.

8.2.2 Evolutionary metaheuristics

Doerner et al. [5] considered a location-routing problem in health care management. In their problem, a closed tour with stops selected from a given set of population nodes has to be found. Not every node needs to be part of the tour. Tours are evaluated according to three criteria which form the objectives of the multi-objective problem.

1. The first objective is to determine an economic efficiency criterion related to the tour length. The effectiveness of workforce employment is measured by the ratio between medical working time and total working time, including travel time and facility setup time.
2. The criterion of average distances to the nearest tour stops is the second objective. Average accessibility is measured by a low average time required by the inhabitants of the considered region to reach the nearest tour stop or the nearest stationary facility.
3. The third objective is the coverage criterion measuring the percentage of the population unable to reach a tour stop within a predefined maximum distance.

To compute approximation to the set of Pareto-efficient solutions, they proposed a Pareto ant colony optimization (P-ACO) technique as well as two GAs, namely the vector evaluated GA (VEGA) and the multi-objective GA (MOGA). P-ACO extends on the standard ACO in three key ways, namely introducing an outer iteration where random weights are assigning to each objective function, incorporating a mechanism to verify whether a newly discovered solution is non-dominated with respect to the current set of candidate solutions and vice versa, and employing a more sophisticated method for managing pheromone levels. VEGA divides the population into three equal parts, each associated with one of the objective functions. From each fraction, a new subset of solutions is selected using the standard GA selection process based on the corresponding objective function. These subsets are then merged and shuffled to form a new population of the original size. A key distinction between VEGA and MOGA is that MOGA uses a specialized ranking mechanism designed to prevent the emergence of solution groups that perform well on only one objective but poorly on the others.

8.3 Optimization approaches using exact algorithms

Vehicle routing problems are presented in Section 8.3.1 while studies in which the considered problem is formulated as a set-packing or set-partitioning model, may be found in §8.3.2.

8.3.1 Vehicle routing problems

Three papers [3, 11, 14] that study vehicle routing problems in which exact algorithms are implemented as solution techniques, are summarized. Providing adequate primary health care in developing countries is often challenging due to the need to balance the number of facilities. Enough facilities need to be ensured to maintain geographic accessibility while keeping them sufficiently staffed and supplied. In many low-income regions, this challenge is worsened by long rainy seasons, during which travel is restricted to paved roads. To address this issue in Ghana's Suhum District, the study by Hodgson et al. [11] applies a covering tour model that explores the use of mobile facilities, aiming to minimize travel distances while ensuring all population centers are served within reach of feasible stops. To solve the problem, the authors applied an exact algorithm developed by Gendreau et al. [9].

Salman et al. [14] considered health service delivery to Syrian seasonal migrant farm workers in Turkey by means of mobile clinics. Their approach to modeling is multi-depot selective routing with limits on tour distance and visit frequencies for multiple service types, *e.g.* vaccination, screening, gynecology and health education, over a finite planning horizon. Multiple clinics departing from their depots visit the selected stop points. A demand point is covered for a service type when it is located within an acceptable distance from at least one clinic's stop point where it can serve its demand. Demand is assumed to emanate from migrant workers in agriculture, but it is not explicitly modeled. It is assumed that a stop near enough to demand points is sufficient to cover all demand with no capacity limit.

They presented a binary integer programming formulation with three objective functions, namely the maximization of the total demand covered, minimization of the total number of clinics while covering predetermined levels of demand and the minimization of the total travel distance while covered predetermined levels of demand and number of clinics. They used the augmented ϵ-constraint method to explore the trade-off between two key objectives, namely maximizing the total coverage and minimizing the number of clinics used. This approach transforms one objective into a constraint and systematically reduces its bound (ϵ) to generate a set of non-dominated, Pareto-efficient solutions. By assigning a 1% decrement in total coverage and a 0.01 multiplier on the surplus of the coverage constraint, they obtained efficient trade-offs over a one-month planning horizon.

Callaghan et al. [3] focused on improving the implementation of mobile health services through a case study conducted in the Witzenberg region of South Africa, where mobile clinics are deployed to serve remove rural communities. The study not only addresses the operational challenge of assigning villages or farms to mobile clinics and determining their monthly visit schedules, but also highlights the importance of cross-disciplinary collaboration between operations researchers and health care professionals. To increase the likelihood of successful implementation, the authors proposed a three-phased mixed-methods modeling framework that integrates practical health care considerations, such as workload balance and fairness, with classical operational objectives like minimization of travel distance.

In the first phase of the authors' framework, both qualitative and quantitative data are collected using qualitative research methods to understand the context and needs of the health care system. In the second phase, a three-stage optimization model is developed. In the first stage, a multi-depot VRP is solved using a branch-cut-and-price algorithm to determine the set of daily clinic routes. In the second stage, a knapsack problem is solved using a branch-and-bound method in an exact solver to fairly allocate the routes among the clinics. In the third stage, a customize routing model constructs monthly schedules that minimize the travel distance between the last stop of one day and the first stop of the next, particularly relevant when clinics must return to the same location. Finally, in the third phase of the authors' framework, the analytic hierarchy process is used in collaboration with key decision makers to evaluate and select their preferred solution from among the modeled alternatives.

8.3.2 Set-packing and set-partitioning formulations

Two papers [1, 10] that incorporate set-packing and set-partitioning formulations, are presented. Long-distance truck drivers in Sub-Saharan Africa face high risks of HIV and other infectious diseases. To address this, the non-governmental organization called North Star Alliance deploys roadside wellness centers (RWCs) at key truck stops. As their network expands, ensuring equitable health care access across all routes becomes increasingly important. Ares et al. [1] addressed the challenge of optimally locating a fixed number of RWCs by balancing effectiveness and equity. The authors proposed a novel set-partitioning model and a column generation algorithm with advanced techniques, achieving near-optimal solutions for large instances and demonstrating that both objectives may be satisfied simultaneously.

The incorporated techniques include dual stabilization, column pool management and accelerated pricing.

- Column generation frequently exhibits significant fluctuations in dual variable values between iterations, which can cause slow convergence and lead to degeneracy. To address this, various techniques have been developed to stabilize the dual variables and, in turn, speed up the convergence process.
- Every generated column is stored in a column pool. Before each iteration, each column in the pool is evaluated to decide whether it should be included in the restricted master problem (RMP). The goal is to balance between maximizing the RMP's objective of including as many promising columns as possible and to reduce the RMP's solution time by limiting the number of columns selected.
- Rather than solving the pricing problem to optimality, accelerated pricing aims to rapidly find a promising, though not necessarily optimal, variable by solving a series of shortest path problems.

Santa González et al. [10] provided a set-packing formulation for the tactical planning of mobile clinic deployment, which is considered to be a multi-period location-routing problem (MLRP). The proposed model aims to select communities to be served and design routes to be performed such that health benefits, measured by means of coverage and continuity of care functions, are maximized throughout the planning horizon. The total benefit of providing health care services is maximized while taking into account a budget for the operational and logistics expenses. Continuity of care is maximized by ensuring that an individual is visited multiple times by the mobile clinics. The authors assessed the value of continuity of care by testing functions with different patterns of the marginal benefits offered by additional visits.

The authors propose a route-generation algorithm which generates a predefined set of feasible routes to be used as input for solving th MLRP using an exact algorithm. The process algorithm requires an estimate of the maximum number of villages a route may cover, calculated based on the available time after serving 50 patients and the setup time per village. The algorithm systemically generates all possible routes that start and end at specified depots, initializing relevant route data and computing associated parameters such as total duration, distance and cost. The duration accounts for village setup times, patient service times and travel time while the cost is calculated as a linear function of distance. Only routes whose total duration does not exceed the allowed time limit are included in the final set of feasible routes.

8.4 Conclusions

Despite this rich theory, several research gaps remain in solving more complex variants of vehicle routing problems, specifically related to multi-depot, capacitated, periodic, stochastic problems, and those that consider additional objectives such as fairness. These problems are NP-hard to solve, and for larger instances, require the use of metaheuristics.

Bibliography VIII

[1] José N. Ares, Harwin De Vries, and Dennis Huisman. "A column generation approach for locating roadside clinics in Africa based on effectiveness and equity". In: *European Journal of Operational Research* 254.3 (2016), pp. 1002–1016. doi: 10.1016/j.ejor.2016.04.031.

[2] O. Baturhan Bayraktar, Dilek Günneç, F. Sibel Salman, and Eda Yücel. "Relief aid provision to en route refugees: Multi-period mobile facility location with mobile demand". In: *European Journal of Operational Research* 301.2 (2022), pp. 708–725. doi: 10.1016/j.ejor.2021.11.011.

[3] Hannah J. Callaghan, Linke Potgieter, and Nadia Le Roux. "Mobile clinics routing and scheduling in the Witzenberg region of South Africa". In: *PLOS One* 20.1 (2025), pp. 1–26. doi: 10.1371/journal.pone.0310086.

[4] Jérémy Decerle, Olivier Grunder, Amir H. El Hassani, and Oussama Barakat. "A hybrid memetic-ant colony optimization algorithm for the home health care problem with time window, synchronization and working time balancing". In: *Swarm and Evolutionary Computation* 46.1 (2019), pp. 171–183. doi: 10.1016/j.swevo.2019.02.009.

[5] Karl Doerner, Axel Focke, and Walter J. Gutjahr. "Multicriteria tour planning for mobile healthcare facilities in a developing country, synchronization and working time balancing". In: *European Journal of Operational Research* 179.3 (2007), pp. 1078–1096. doi: 10.1016/j.ejor.2005.10.067.

[6] Amir M. Fathollahi-Fard, Abbas Ahmadi, and Behrooz Karimi. "Multi-objective optimization of home healthcare with working-time balancing and care continuity". In: *Sustainability* 13.2 (2021), pp. 1–33. doi: 10.3390/su132212431.

[7] Sana Frifita and Malek Masmoudi. "VNS methods for home care routing and scheduling problem with temporal dependencies, and multiple structures and specialties". In: *International Transactions in Operational Research* 27.1 (2020), pp. 291–313. doi: 10.1111/itor.12604.

[8] Yaping Fu, Xiaomeng Ma, Kaizhou Gao, Hongfeng Wang, Ali Sadollah, and Lyuyan Chen. "Multi-objective migrating birds optimization for solving stochastic home health care routing and scheduling problems considering caregiver working time constraints". In: *Swarm and Evolutionary Computation* 85.1 (2024), pp. 1–19. doi: 10.1016/j.swevo.2024.101484.

[9] Michel Gendreau, Gilbert Laporte, and Frédéric Semet. "The covering tour problem". In: *Operations Research* 45.1 (1997), pp. 568–576. doi: 10.1287/opre.45.4.568.

[10] Rosemarie Santa González, Marilène Cherkesly, Teodor Gabriel Crainic, and Marie-Ève Rancourt. "Multi-period location routing: An application to the planning of mobile clinic operations in Iraq". In: *Computers & Operations Research* 159 (2023), paper no. 106288. doi: 10.1016/j.cor.2023.106288.

[11] M. John Hodgson, Gilbert Laporte, and Frédéric Semet. "A covering tour model for planning mobile health care facilities in Suhum District, Ghama". In: *Journal of Regional Science* 38.4 (1998), pp. 621–638. doi: 10.1111/0022-4146.00113.

[12] Wenheng Liu, Mahjoub Dridib, Amir M Fathollahi-Fard, and Amir Hajjam El Hassani. "A customized adaptive large neighborhood search algorithm for solving a multi-objective home health care problem in a pandemic environment". In: *Swarm and Evolutionary Computation* 86.1 (2024), paper no. 101507. doi: 10.1016/j.swevo.2024.101507.

[13] Andrés Arias Londoño, Walter Gil González, Oscar Danilo Montoya Giraldo, and John Wilmer Escobar. "A hybrid heuristic approach for the multi-objective multi depot vehicle routing problem". In: *International Journal of Industrial Engineering Computations* 15.1 (2024), pp. 337–354. doi: 10.5267/j.ijiec.2023.9.006.

[14] F. Sibel Salman, Eda Yücel, İlker Kayı, Sedef Turper-Alışık, and Abdullah Coşkun. "Modeling mobile health service delivery to Syrian migrant farm workers using call record data". In: *Socio-Economic Planning Sciences* 77.1 (2021), pp. 1–17. doi: 10.1016/j.seps.2020.101005.

[15] Sinem K Savaşer and Bahar Y Kara. "Mobile healthcare services in rural areas: an application with periodic location routing problem". In: *OR Spectrum* 44.3 (2022), pp. 875–910. doi: 10.1007/s00291-022-00670-3.

[16] Yong Shi, Toufik Boudouh, and Olivier Grunder. "A robust optimization for a home health care routing and scheduling problem with consideration of uncertain travel and service times". In: *Transportation Research Part E: Logistics and Transportation Review* 128.1 (2019), pp. 52–95. doi: 10.1016/j.tre.2019.05.015.

[17] Adria Soriano, Margaretha Gansterer, and Richard F Hartl. "The multi-depot vehicle routing problem with profit fairness". In: *International Journal of Production Economics* 255.1 (2023), pp. 1–13. doi: 10.1016/j.ijpe.2022.108669.

[18] Jiao Zhao, Tao Wang, and Thibaud Monteiro. "A bi-objective home health care routing and scheduling problem under uncertainty". In: *International Journal of Environmental Research and Public Health* 21.3 (2024), pp. 1–27. doi: 10.3390/ijerph21030377.

Chapter IX

Patrolling Streets

Private security companies provide rapid alarm response and visible patrol services, complementing police forces while competing for clients through reliability and speed. This chapter focuses on the armed response problem, which focuses on allocating and routing patrol cars to balance three goals: minimizing travel distances (cost efficiency), ensuring equitable distribution of vehicles (client trust), and reducing response times (service quality). The study frames armed response within the broader literature on police patrol and emergency service optimization. We review related approaches across four domains: district design, resource allocation, patrol route design, and dynamic relocation strategies. By linking private security practices to established operations research and graph-theoretic methods, this chapter highlights opportunities for efficient, fair, and client-centered policies.

Keywords: Resource allocation, call for service, response time, redeployment policy, dynamic relocation.

IX

The Armed Response Problem: Optimizing Private Security Patrols

Nina Chiarelli, University of Primorska, Slovenia
Clément Dallard, University of Fribourg, Switzerland
Tınaz Ekim, Boğaziçi University, Türkiye
Hanna Furmańczyk, University of Gdańsk, Poland
J. Pascal Gollin, University of Primorska, Slovenia
Claire Hilaire, University of Primorska, Slovenia
Martin Milanič, University of Primorska, Slovenia
Peter Muršič, University of Primorska, Slovenia
Linke Potgieter, Stellenbosch University
Adriana Roux, Stellenbosch University, South Africa

Private security companies play an increasingly important role in urban safety, as an added layer of security that supports police patrolling. They offer rapid alarm response services to paying clients alongside visible street patrols. Their dual mission – minimizing response times while maintaining a reassuring public presence – poses a significant operational challenge. In this chapter, mathematical approaches to address this challenge are presented.

The search for mathematical solutions to minimize response times and determine efficient vehicle locations is motivated by the private security context in South Africa, where numerous companies provide round-the-clock patrol and monitoring services with trained teams ready to respond to alerts and dispatch emergency assistance. These companies offer an additional layer of support to paying clients, independent from police patrolling services that serve the broader public. Clients have alarms installed (as a security measure against theft) in their homes or offices. When an alarm is triggered, the security company must dispatch a patrol car as quickly as possible to assess the situation. Competition among companies makes rapid response times a key differentiator. Additionally, patrol cars frequently circulate near clients even when not responding to alarms, enhancing perceived reliability through the visible presence of security personnel.

The armed response problem considered in this chapter involves assigning a set of patrol cars (guards) to a set of clients, ensuring that each client receives adequate security service. The objectives are to minimize total distance covered over all patrol cars (from a company cost perspective), maximize an equitable distribution of patrol cars across the network of clients (from a client perception perspective), and minimize response time should an alarm be triggered in the network (from a client experience perspective). The problem also involves assigning positions to patrol cars so that, at any time, an available vehicle can reach a house experiencing an alarm within a short response time. Vehicles responding to an alarm are considered unavailable until the situation is resolved, and those not responding typically patrol the streets and neighborhoods regularly, both to maintain coverage and to enhance the perceived presence of security.

Many problems related to the armed response problem have been studied in the literature. There is a vast literature on police patrolling problems; see, e.g., [30], where a systematic overview is provided from the practice of Operations Research point of view. In [30], the problem is divided into three separate stages:

(a) District Design,
(b) Resource Allocation,
(c) Route Design,

and for each of them, several criteria (cost, time, etc.) are considered along with the different possibilities for the models.

The district design problem is first reviewed in §9.1, which includes graph partitioning problems. Resource allocation models are then reviewed in §9.2, which includes set covering approaches, facility location models, and location-routing models. In §9.3 research done on patrol route design is considered. Finally, in §9.4, the relocation problem is considered wherein, once a response vehicle is assigned to an alarm, all remaining vehicles should be relocated to ensure that all coverage constraints are met.

9.1 District design

District design (also referred to as police districting, beat design, or sectoring) concerns the partitioning of a service region into contiguous territorial units assigned to individual response agents or patrol vehicles. Well-designed districts materially affect workload balance, visibility, and response performance, and thus constitute a fundamental first stage in the armed response pipeline [25].

Typical objectives and constraints in districting models include: equitable distribution of expected demand or workload across districts; bounds on travel time or response time from any demand point to its assigned resource; contiguity/connectedness of each district; geometric compactness or limits on shape irregularity; and size constraints measured in area, population, or client counts [25]. Practical districting must therefore trade off several often competing goals (equity, service level, and geographic regularity) while respecting administrative or operational constraints.

A variety of algorithmic approaches appear in the literature. Metaheuristics such as simulated annealing and other neighbourhood-search methods have been successfully used to explore the combinatorial space of contiguous partitions and to enforce shape or size constraints while optimizing workload and response metrics [11]. Complementary approaches use mixed-integer programming and covering/location formulations to capture explicit service constraints: for instance, maximal covering and backup/backup-coverage formulations enable designers to impose explicit response-time guarantees and to ensure redundancy where needed [10]. In practice, these optimization techniques are frequently combined with GIS tools and empirical travel-time estimates to ensure that candidate partitions translate into desirable operational performance in the field [25].

Empirical and computational findings reported in the literature reveal common patterns: allowing moderate flexibility in district shape can yield substantial improvements in workload balance or coverage without large increases in response time, whereas very strict geometric constraints (strong compactness or convexity requirements) may force trade-offs that degrade performance on service measures. The choice of atomic unit for partitioning (census blocks, reporting districts, street segments) also matters: finer granularity increases modeling fidelity but raises computational cost and data requirements [11, 25].

In addition to applied research on police districting and patrol planning, there exists a substantial body of more theoretical work on graph partitioning. In these studies, the practical problem of dividing a service area is abstracted to a formal graph-theoretic setting, where the goal is to partition a graph into subgraphs that satisfy specific constraints, such as bounded diameter, connectivity, or compactness. These works provide fundamental insights into the computational complexity, algorithmic strategies, and approximation approaches for partitioning graphs according to defined parameters, establishing a bridge between combinatorial graph theory and real-world applications like emergency response or patrol design. For example, the paper [11] models police district design as a graph partitioning problem, where a city or service area is represented as a network of nodes (e.g., intersections or neighborhoods) and edges (streets or travel paths). The goal is to divide the graph into patrol districts such that each district is contiguous, compact, and balanced in workload, while also minimizing travel times and response times within districts. The authors formulate this as an optimization problem and apply simulated annealing to find high-quality partitions. In essence, the work explicitly treats district design as a bounded-diameter partitioning problem, where the "diameter" reflects travel or response distance, linking theoretical graph partitioning concepts directly to practical police patrol planning. The paper [8] investigates the computational complexity of partitioning a graph into clusters with a specified maximum diameter. The authors explore the decision problem of determining whether a given graph can be partitioned into clusters such that each cluster has a diameter not exceeding a specified value. They analyze the problem's complexity and provide insights into its computational challenges. The paper [14] studies clustering problems in graphs where each cluster must be connected and the goal is to keep clusters compact, either by minimizing the maximum distance to a cluster center or the maximum distance between any two points in a cluster. The authors develop approximation algorithms for these problems, analyze their computational complexity, and provide improved results for special types of metrics, such as Euclidean spaces. In the context of graph partitioning, the relevance of this work stems from the fact that it explicitly addresses partitioning a graph into connected subgraphs with bounded diameter, which is analogous to designing patrol regions or response zones where connectivity and travel distance constraints are important.

For the armed response problem specifically, district design must be considered together with vehicle routing and dynamic relocation. Unlike many static districting models that assume resource availability, armed response settings must account for temporary unavailability of vehicles while they attend incidents; this calls for districting objectives that are robust to temporary reductions in available coverage and that facilitate efficient on-the-fly reallocation or relocation of remaining vehicles.

9.2 Resource allocation

With regard to the resource allocation stage, problems based on time, cost, unpredictability, and workload variation were considered in [30]. All variants of the resource allocation stage have a vast literature, considering mathematical programming models (see, e.g., [1, 2, 23, 24, 26, 27, 35]), game theoretic models (see, e.g., [4, 22, 28, 29, 34, 36]), stochastic models (see, e.g., [5, 7]), as well as simulation models (see, e.g., [17, 23, 33]).

In mathematical programming models, the objectives range from minimizing response time and travel distance to maximizing visibility and coverage.

Location-allocation models for traffic control vehicles are investigated in [1] by Adler et al. They present various IP problem formulations that ensure full coverage of the network while maximizing police presence. They applied their models to a real case study of the interurban road network of Northern Israel and compared the results to the classical models of MCLP (Maximum Covering Location Problem) and SCP (Set Covering Problem).

The scheduling of a fixed number of security patrols in a rail network is studied in [24]. They propose a mathematical programming model that maximizes coverage while minimizing total travel distance, and validate their approach on the Singapore rail network.

In [26] the probability that no response vehicle arrives at the problematic spot within the required time is minimized, thereby accounting for vehicle response times. Real-life data from a Belgian police zone is used to test their models.

Few studies have examined these variants from a graph-theoretical perspective. This representation establishes links to a broad range of algorithmic graph problems, including shortest paths [13], network flows [3, 16], and domination problems [19, 20, 21], among others. For example, Blair et al. [6] introduce and analyze graph-theoretic models for placing emergency responders to effectively manage multiple simultaneous incidents. The authors define special sets of vertices that ensure coverage when emergencies arise and analyze the size of such sets across different graph types. In addition, they develop bounds, compute exact values for several families of graphs, and connect these ideas to well-known concepts in domination theory.

9.3 Route design

Dewinter et al. [12] develop a police patrol algorithm that integrates proactive and reactive patrolling. They employ a discrete-event simulation model to compare a p-median redeployment strategy (solved via integer programming) with several benchmark strategies.

Willemse et al. [31] show that finding optimal patrol routes for guards in a gated community can be formulated as a min-max k-Rural Postman Problem. They propose a tabu search algorithm to solve this problem and apply it to the Midfield Estate in Gauteng, South Africa.

Some graph-theoretical models make use of spanning trees and Hamiltonian cycles to find optimal patrol routes in a network. For instance, to identify important patrol routes that account for both the significance of locations and the network topology, Chawathe [9] presents a dynamic programming approach that leverages the graph's spanning tree to find high-density routes.

In [15] patrol paths for a group of mobile robots in a fixed area are determined by the construction of Hamiltonian cycles and instructing robots to move along the cycle maintaining their distance from each other. This method ensures that all points are visited with the same frequency.

9.4 Relocation

In addition to the allocation problem, in [32] Yıldırım and Soylu considered the placement and relocation of emergency vehicles in a district-based network to ensure multiple coverage of the entire network. A fixed set of emergency vehicles is distributed through a network ensuring that all districts in the network are covered to a district-based coverage requirement. A dispatcher will inform an emergency vehicle to attend to an alarm and will then make use of a compliance table to relocate the remaining emergency vehicles to ensure that all coverage constraints are met.

The maximal expected coverage relocation problem (MECRP) was modified by Gendreau et al. in [18] to incorporate dynamic relocation and multiple coverage requirements. A network flow model and a heuristic are used to obtain the relocation plan. The paper further implements their models for natural gas emergency services in the region of Kayseri, Turkey, and shows through a simulation study that their model improves on the previous static models in the literature.

9.5 Conclusions

The armed response problem captures a practical and timely challenge faced by private security companies: how to balance rapid response to alarms with the need for continuous, visible patrolling. It is a multi-faceted problem that includes consideration for district design, allocation of response vehicles, efficient patrol route design, as well as the relocation of response vehicles when an alarm is triggered. Literature indicates that future research should explore more graph-theoretic approaches, especially to the allocation problem on a network, as well as simulation to evaluate strategies determined by graph-theoretic approaches, under realistic, stochastic demand scenarios, thereby bridging the gap between theoretical models and operational deployment.

[1] Nicole Adler, Alfred Shalom Hakkert, Jonathan Kornbluth, Tal Raviv, and Mali Sher. "Location-allocation models for traffic police patrol vehicles on an interurban network". In: *Annals of Operations Research* 221.1 (2014), pp. 9–31. doi: 10.1007/s10479-012-1275-2.

[2] Nicole Adler, Alfred Shalom Hakkert, Tal Raviv, and Mali Sher. "The traffic police location and schedule assignment problem". In: *Journal of Multi-Criteria Decision Analysis* 21.5-6 (2014), pp. 315–333. doi: 10.1002/mcda.1522.

[3] Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin. *Network Flows — Theory, Algorithms and Applications*. Prentice Hall, Inc., 1993, pp. xvi+846.

[4] Steve Alpern, Alec Morton, and Katerina Papadaki. "Patrolling games". In: *Operations Research* 59.5 (2011), pp. 1246–1257. doi: 10.1287/opre.1110.0983.

[5] Mukhopadhyay Ayan, Zhang Chao, Vorobeychik Yevgeniy, Tambe Milind, Pence Kenneth, and Speer Paul. "Optimal allocation of police patrol resources using a continuous-time crime model". In: *International Conference on Decision and Game Theory for Security*. 2016, pp. 139–158. doi: 10.1007/978-3-319-47413-7_9.

[6] Jean Blair, Wayne Goddard, Sandra M Hedetniemi, Stephen T. Hedetniemi, Fredrik Manne, and Douglas Rall. "Emergency response sets in graphs". In: *Journal of Combinatorial Mathematics and Combinatorial Computing* 68 (2009), pp. 225–243. url: https://combinatorialpress.com/article/jcmcc/Volume%2068/vol-068-paper%2016.pdf (visited on 09/29/2025).

[7] Matthew Brown, Sandhya Saisubramanian, Pradeep Varakantham, and Milind Tambe. "STREETS: Game-theoretic traffic patrolling with exploration and exploitation". In: *Proceedings of the AAAI Conference on Artificial Intelligence*. Vol. 28. 2. 2014, pp. 2966–2971. doi: 10.1609/aaai.v28i2.19028.

[8] Jou-Ming Chang, Jinn-Shyong Yang, and Sheng-Lung Peng. "On the complexity of graph clustering with bounded diameter". In: *Proceedings of the 2014 International Computer Science and Engineering Conference (ICSEC)*. IEEE, 2014, pp. 18–22. doi: 10.1109/ICSEC.2014.6978122.

[9] Sudarshan S. Chawathe. "Organizing hot-spot police patrol routes". In: *IEEE Intelligence and Security Informatics*. 2007, pp. 79–86. doi: 10.1109/ISI.2007.379538.

[10] Kevin M Curtin, Karen Hayslett-McCall, and Fang Qiu. "Determining optimal police patrol areas with maximal covering and backup covering location models". In: *Networks and Spatial Economics* 10.1 (2010), pp. 125–145. doi: 10.1007/s11067-007-9035-6.

[11] Steven J D'Amico, Shoou-Jiun Wang, Rajan Batta, and Christopher M Rump. "A simulated annealing approach to police district design". In: *Computers and Operations Research* 29.6 (2002), pp. 667–684. doi: 10.1016/S0305-0548(01)00056-9.

[12] Maite Dewinter, Caroline Jagtenberg, Christophe Vandeviver, Philipp M. Dau, Tom Vander Beken, and Frank Witlox. "Reducing police response times: optimization and simulation of everyday police patrol". In: *Networks* 84.3 (2024), pp. 363–381. doi: 10.1002/net.22241.

[13] Edsger Wybe Dijkstra. "A note on two problems in connexion with graphs". In: *Numerische Mathematik* 1 (1959), pp. 269–271. doi: 10.1007/BF01386390.

[14] Lukas Drexler, Jan Eube, Kelin Luo, Dorian Reineccius, Heiko Röglin, Melanie Schmidt, and Julian Wargalla. "Connected *k*-center and *k*-diameter clustering". In: *Algorithmica* 86.11 (2024), pp. 3425–3464. doi: 10.1007/s00453-024-01266-9.

[15] Yehuda Elmaliach, Noa Agmon, and Gal A. Kaminka. "Multi-robot area patrol under frequency constraints". In: *Annals of Mathematics and Artificial Intelligence* 57.3 (2009), pp. 293–320. doi: 10.1007/s10472-010-9193-y.

[16] Lester Randolph Ford Jr. and Delbert Ray Fulkerson. *Flows in Networks*. Princeton University Press, 1962, pp. xii+194.

[17] Vasco Furtado and Eurico Vasconcelos. "A multiagent simulator for teaching police allocation". In: *AI Magazine* 27.3 (2006), pp. 63–74. doi: 10.1609/aimag.v27i3.1893.

[18] Michel Gendreau, Gilbert Laporte, and Fréëric Semet. "Solving an ambulance location model by tabu search". In: *Location Science* 5.2 (1997), pp. 75–88. doi: 10.1016/S0966-8349(97)00015-6.

[19] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning, eds. *Topics in Domination in Graphs*. Vol. 64. Developments in Mathematics. Springer, 2020, pp. viii+545. doi: 10.1007/978-3-030-51117-3.

[20] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning, eds. *Structures of Domination in Graphs*. Vol. 66. Developments in Mathematics. Springer, 2021, pp. viii+536. doi: 10.1007/978-3-030-58892-2.

[21] Teresa W. Haynes, Stephen T. Hedetniemi, and Michael A. Henning. *Domination in Graphs — Core Concepts*. Springer Monographs in Mathematics. Springer, 2023, pp. xx+644. doi: 10.1007/978-3-031-09496-5.

[22] Christopher Kiekintveld, Manish Jain, Jason Tsai, James Pita, Fernando Fernando Ordónez, and Milind Tambe. "Computing optimal randomized resource allocations for massive security games". In: *Proceedings of the 8th International Conference on Autonomous Agents and Multiagent Systems*. 2009, pp. 689–696. url: https://projects.iq.harvard.edu/files/teamcore/files/2009_7_teamcore_computing_aamas_09.pdf (visited on 09/26/2025).

[23] Henry C.W. Lau, George T.S. Ho, Yi Zhao, and W.T. Hon. "Optimizing patrol force deployment using a genetic algorithm". In: *Expert Systems with Applications* 37 (2010), pp. 8148–8154. doi: 10.1016/j.eswa.2010.05.073.

[24] Hoong Chuin Lau, Zhi Yuan, and Aldy Gunawan. "Patrol scheduling in urban rail network". In: *Annals of Operations Research* 239.1 (2016), pp. 317–342. doi: 10.1007/s10479-014-1648-9.

[25] Federico Liberatore, Miguel Camacho-Collados, and Begoña Vitoriano. "Police districting problem: literature review and annotated bibliography". In: *Optimal Districting and Territory Design* (2020), pp. 9–29. doi: 10.1007/978-3-030-34312-5_2.

[26] Mark Moonen, Dirk Cattrysse, and Dirk Van Oudheusden. "Organising patrol deployment against violent crimes". In: *Operational Research* 7.3 (2007), pp. 401–417. doi: 10.1007/BF03024855.

[27] Mohammed Muaafa and Jose Emmanuel Ramirez-Marquez. "Engineering management models for urban security". In: *IEEE Transactions on Engineering Management* 64.1 (2016), pp. 29–41. doi: 10.1109/TEM.2016.2634119.

[28] Katerina Papadaki, Steve Alpern, Thomas Lidbetter, and Alec Morton. "Patrolling a border". In: *Operations Research* 64.6 (2016), pp. 1256–1269. doi: 10.1287/opre.2016.1511.

[29] James Pita, Manish Jain, Janusz Marecki, Fernando Ordóñez, Christopher Portway, Milind Tambe, Craig Western, Praveen Paruchuri, and Sarit Kraus. “Deployed ARMOR protection: the application of a game theoretic model for security at the los angeles international airport”. In: *Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems: industrial track*. 2008, pp. 125–132. doi: 10.5555/1402795.1402819.

[30] Sukanya Samanta, Goutam Sen, and Soumya Kanti Ghosh. “A literature review on police patrolling problems”. In: *Annals of Operations Research* 316.2 (2022), pp. 1063–1106. doi: 10.1007/s10479-021-04167-0.

[31] Elias J. Willemse and Johan W. Joubert. “Applying min-max *k* postmen problems to the routing of security guards”. In: *Journal of the Operational Research Society* 63.2 (2012), pp. 245–260. doi: 10.1057/jors.2011.26.

[32] Betül Yildirim and Banu Soylu. “Relocating emergency service vehicles with multiple coverage and critical levels partition”. In: *Computers & Industrial Engineering* 177 (2023), paper no. 109016. doi: 10.1016/j.cie.2023.109016.

[33] Chao Zhang, Victor Bucarey, Ayan Mukhopadhyay, Arunesh Sinha, Yundi Qian, Yevgeniy Vorobeychik, and Milind Tambe. “Using abstractions to solve opportunistic crime security games at scale”. In: *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems*. 2016, pp. 196–204. doi: 10.5555/2936924.2936956.

[34] Yue Zhang and Donald Brown. “Simulation optimization of police patrol districting plans using response surfaces”. In: *Simulation* 90.6 (2014), pp. 687–705. doi: 10.1177/0037549714533159.

[35] Ke Quan Zhao, Li Luo, and Yuan Mei Xia. “The optimal dispatch of traffic and patrol police service platforms”. In: *Journal of Applied Mathematics* 2012.1 (2012), paper no. 292415. doi: 10.1155/2012/292415.

[36] Noemí Zoroa, M José Fernández-Sáez, and Procopio Zoroa. “Patrolling a perimeter”. In: *European Journal of Operational Research* 222.3 (2012), pp. 571–582. doi: 10.1287/opre.2016.1511.

Chapter X

Electric Power Systems

This chapter explores how graph theory and optimization guide the planning and operation of modern power systems. By modeling the grid as a network, graph theory reveals its hidden structure, enabling clustering, reliability assessment, and efficient distribution through tools like minimum spanning trees and layering-pruning. Optimization techniques complement this perspective, using linear programming and heuristics to schedule generation, position substations, and place renewable resources where they can do the most good. Learning these approaches is essential, not only to keep the lights on, but to create grids that are cleaner, more resilient, and able to support a future powered by sustainable energy.

Keywords: Electricity grids, graph theory, optimization, renewable energy, reliability.

X

Electric Power Systems

Michael Ayomoh, University of Pretoria, South Africa
Miklós Krész, University of Primorska, Slovenia
Iztok Peterin, University of Maribor, Slovenia
Valisoa Rakotonarivo, University of Pretoria, South Africa

South Africa is currently facing significant challenges in its electricity sector. Energy supply has become increasingly constrained, while demand for electricity continues to rise due to urbanization, industrial growth, and population increases. This imbalance has led to frequent load shedding, high costs, and concerns over the long-term sustainability of the national grid.

To address these issues, there is a growing need to optimize the existing electricity distribution network and integrate alternative energy sources, including renewable generation such as solar, wind.

A variety of methodological approaches have been developed to support the optimization of electricity systems. Among the most widely applied are:

- Graph-Theoretic Models: By representing the power grid as a network of nodes (buses/hubs) and edges (transmission lines), graph theory facilitates the identification of critical components, analysis of network connectivity, and optimization of power flows. Applications include vulnerability assessment, reconfiguration strategies to minimize losses, and enhancement of fault tolerance.
- Mathematical Optimization Models: Linear programming, mixed-integer programming, and stochastic optimization methods are used to minimize generation and transmission costs, improve network efficiency, and guide capacity expansion planning under demand and policy constraints.
- Machine Learning and Data-Driven Approaches: Predictive analytics and reinforcement learning are increasingly applied to electricity systems for demand forecasting, renewable output prediction, and dynamic grid optimization. Accurate forecasting enables better generator scheduling, improved use of storage resources, and early fault detection, thereby enhancing both efficiency and reliability.

10.1 Graph theory-based approaches

The application of graph theory in power systems has emerged as a powerful approach to address the increasing complexity of modern electricity distribution networks. Graph-theoretic techniques allow system components, such as generators, loads, and control elements, to be represented as nodes, while the transmission lines connecting them are represented as edges. Formally, a power system can be expressed as a graph $G = (V, E)$, where V denotes the set of nodes and E represents the set of edges. This abstraction provides a mathematically tractable framework for analyzing structural, operational, and stability properties of power systems.

10.1.1 Transmission networks

Ishizaki et al. [8] provide a comprehensive overview of graph-theoretic applications in power system modeling, dynamics, coherency, and control. Their work emphasizes that understanding the interactions between nodes, rather than just the network topology, is crucial. Focusing primarily on transmission-level models, they analyze a linearized swing model and demonstrate that generators and bus voltages exhibit dynamically cohesive and synchronized behavior when the network topology and physical parameters possess certain symmetries. By leveraging graph automorphisms, they characterize dynamic synchronism and apply it to aggregate generator states and bus voltages through network clustering. The aggregated differential-algebraic equations preserve a weighted graph Laplacian structure, maintaining Kirchhoff's laws, and provide a physically interpretable model reduction approach compared to traditional techniques.

10.1.2 Distribution networks

Several graph-theoretic methods have been proposed to optimize energy distribution networks in terms of cost and efficiency. One classical approach involves the use of a minimum spanning tree (MST) to design the layout of distribution lines. In this model described in [16], nodes represent substations, load points, and transformers, while edges correspond to switches, grounding devices, and transmission corridors, with weights representing current flow. By applying algorithms such as Prim's algorithm, the MST identifies the most efficient network layout by minimizing the total pipeline length. However, this method does not account for energy flows or capacity requirements, limiting its practical applicability.

The next study combines dynamic programming with graph theory to address these limitations. In [4] Ding et al. proposed a method that uses kernel density analysis to identify optimal energy station locations based on aggregated building load demands. A 0–1 dynamic programming model, inspired by the knapsack problem, allocates marginal building loads to minimize load fluctuations and leverage complementary load profiles. An improved Prim algorithm is then applied to design the pipeline network, incorporating hydraulic characteristics, investment, and operational costs to ensure economic efficiency and system stability. This approach, demonstrated on a university campus, successfully reduces load fluctuation rates, smooths peak demand, and improves the overall hydraulic balance of the network. However, the methodology focuses primarily on regional-scale systems and economic outcomes, with limited consideration of broader power grid interactions or environmental impacts.

The layering and pruning method (L&PM) proposed by He et al. [7] further integrates capacity optimization. Nodes are layered according to their connectivity to the energy source, and redundant pipelines within the same layer are pruned to minimize excess capacity. For nodes connected to multiple upstream nodes, the shortest pipeline is selected, while multi-source adjustments account for bidirectional energy flows and capacity allocation. Power flows are calculated sequentially from the outer layers to the source, and pipeline capacities are determined with a reliability margin. Case studies, such as in Guangzhou, China, show that L&PM achieves significant cost savings while improving operational reliability through reduced load loss ratios and enhanced energy transmission margins.

10.1.3 Reliability and resilience

Reliability is a critical consideration in distribution system planning, particularly with the rise of distributed generation (different sources of energy).

The reliability and resilience of an electricity distribution system is largely dependent on several internal and external driving factors. In [9], the authors addressed a predictive problem related to the duration of unplanned power outages, with the aid of an historical outage record for the training of a neural network predictor. The duration prediction was conducted using environmental factors, and updated using incoming field reports premised on natural language processing for an automatic analysis of texts. It was further stated that experiments premised on 15 years of power outage records showed good initial results and improved performance taking advantage of texts. Their case studies showed that the language processing identifies phrases that point to power outage causes and the corresponding repair steps.

Sireesha, Rao, and Kumar [13] propose transforming traditional distribution systems into clusters of multiple microgrids (MMGs) using weighted graph partitioning, with weights based on line apparent power. This clustering enables local control, reactive power compensation, loss reduction, and improved reliability. Their graph-based reliability event model quantifies short- and long-term outages, voltage sags, and associated economic losses to customers. By integrating distributed generation and microgrids, the system becomes more resilient to both permanent and momentary faults. Evaluation using standard reliability indices such as SAIFI, SAIDI, CAIDI, and AENS demonstrates that reliability is highly sensitive to the placement, number, and availability of distributed generators. Optimal microgrid clustering can significantly enhance performance, making graph theory a key tool for planners in active distribution networks with high renewable integration.

Complementary work by Gonzales-Sanchez et al. [12] introduces a graph-theory-based fault location method for transmission systems with renewable energy sources. The method models the grid as a graph, with buses as nodes and transmission lines as weighted edges. Fault detection involves measuring voltages and currents to calculate the apparent fault impedance, comparing it with graph edge impedances, and identifying the most likely faulted line and location. While effective, the method relies heavily on accurate system data and has limited validation under real-world complexities.

Together, these studies illustrate that graph theory enables practical optimization in modern distribution systems, particularly to improve reliability under increasing renewable penetration. Graph-based clustering, reliability indices, and connectivity analysis provide a versatile toolkit for planning the transition from passive networks to active, smart grids.

10.1.4 Research gap

Despite these advancements, several challenges remain. The scalability of graph-theoretic methods to large real-world power systems is uncertain, especially considering the dynamic behavior introduced by high levels of renewable integration and distributed generation. In addition, socioeconomic and regulatory factors, such as those present in South Africa, including chronic supply shortages, tariff structures, and regulatory constraints, can influence the feasibility and effectiveness of microgrid clustering strategies. Addressing these gaps is essential to ensure that graph-theoretic solutions are both technically robust and practically implementable in diverse operational environments.

10.2 Optimization techniques

The optimization of electricity systems has become an increasingly important area of research, driven by growing electricity demand, the integration of renewable energy sources, and the need for reliable, cost-effective, and sustainable operation. To address these challenges, a wide range of optimization techniques has been developed and applied, focusing on both the planning and operational stages of power systems. Linear programming methods, for instance, have been used to determine economically optimal generation schedules and resource allocation in power plants [3, 14]. Heuristic and metaheuristic algorithms provide practical solutions to complex, large-scale planning problems [11] where exact methods may be computationally infeasible. Across these methods, the common goal is to enhance efficiency, reduce costs, and improve the reliability and resilience of electricity networks, laying the foundation for a sustainable and flexible energy system capable of meeting future demand.

10.2.1 Operations research methods

Clack, Xie, and MacDonald [3] present two linear programming (LP) optimization techniques aimed at designing large-scale electrical power systems that integrate renewable energy sources, high-voltage direct current (HVDC) transmission, and energy storage. The first technique focuses on load matching, minimizing the deviation between electricity generation and demand over time, while the second emphasizes cost minimization, optimizing the system's economic efficiency. Both methods model the entire system, including variable and conventional generators, transmission, storage, and demand using LP to ensure computational tractability for large-scale applications. The authors intentionally chose linear programming over non-linear approaches to maintain solution feasibility and computational efficiency, particularly for high-temporal and spatial resolutions. They demonstrate the applicability of these techniques through a sample test case, illustrating their capability to inform the design of sustainable and resilient energy systems.

Similarly, Sukono et al. [14] present a linear programming (LP) model designed to optimize electricity generation at a power plant in Indonesia. The model integrates multiple factors that influence generation efficiency, including fuel consumption, steam production, and turbine electricity output. Using LP, the authors aim to determine the most cost-effective combination of gas and oil fuel usage, steam production, and electricity generation. The optimization process involves three key steps:

1. determining the optimal fuel mix for each boiler unit,
2. calculating the steam production from the boilers,
3. and optimizing electricity production from the turbines using the generated steam.

The mathematical model incorporates various constraints such as production capacities, fuel availability, and operational costs. Using Project-Oriented Modeling (POM) software, the authors demonstrate that the LP model can effectively minimize operational costs while meeting electricity demand. The results highlight the importance of considering efficiency factors and resource allocation in power plant operations to achieve economic optimization. This approach offers valuable insights for improving the economic performance of thermal power plants through systematic optimization techniques.

In [5], a multiobjective optimization model was presented for the planning and management of investments in distributed heat and electricity supply systems. Different measures of energy efficiency were taken into account. This extends to various storage systems, heat and power generation units, and energy-saving upgrade measures. In addition, district heating networks were considered as an alternative conventional individual heat supply for each building. The multi-objective optimization problem was split into three subgroups in a bid to reduce the computational complexity. This enabled a high level of detail in the optimization, while also addressing the comprehensive investigation of districts with more than 100 buildings. The developed model was deployed to a case study district in a medium-sized town in Germany in a bid to conduct analysis on the effects of different efficiency measures regarding total costs and emissions of CO_2 equivalents.

Furthermore, the research in [10], developed a multi-objective energy system such that key variables such as electricity, heat, cooling, fuels, transport, etc. are made to optimally interact as an integrated entity at diverse operational levels including at the district level, city, and region levels. This created an important and unique opportunity to increase all the interacting multifaceted entities including technical, economic, and environmental performance in comparison to a classical energy system whose modeling considerations are treated separately.

Allegrini et al. [1] presented a detailed review of modeling techniques and related software tools that addressed the district-level integrated energy system. Following that buildings play a significant role in urban energy systems with respect to demand and supply of energy, it further asserted that it is largely insufficient to run simulations of buildings using isolation from the microclimate and energy system in which they operate or to model an urban energy system without consideration of the buildings that they serve.

New models and tools that address these district-level interactions are reviewed and their competences assessed. These are divided into the following sections: district energy systems (including heat networks, multi-energy systems (MES), and low-temperature networks), renewable energy generation (including solar, bioenergy, wind, and the related topic of seasonal storage), and the urban microclimate as it relates to energy demands. The scope and detail covered by twenty cross-disciplinary tools is summarized in a matrix; many other tools that focus on specific areas are also discussed. We end by summarizing the current state of district-scale urban energy modeling as it relates to the built environment, along with our perspective on future challenges and research directions.

This performance improvement can take place at both the operational and the planning stage. While such systems and in particular systems with distributed generation of multiple energy vectors DMG (distributed multi-generation) can be a key option to decarbonize the energy sector, the approaches needed to model and relevant tools to analyze them are often of great complexity. Likewise, it is not straightforward to identify performance metrics that are capable of properly capturing costs and benefits that are related to various types of MES according to different criteria. The aim of this invited paper is thus to provide the reader with a comprehensive and critical overview of the latest models and assessment techniques that are currently available to analyze MES and in particular DMG systems, including for instance concepts such as energy hubs, microgrids, and VPPs (virtual power plants), as well as various approaches and criteria for energy, environmental, and techno-economic assessment.

10.2.2 Heuristic and hybrid approaches

Breda and Mestria [11] address the challenge of optimizing the planning of electric power distribution systems, focusing on the strategic placement of substations to enhance service quality and reduce operational costs. They model the problem as a combinatorial optimization task, employing the p-median model to determine optimal substation locations. To solve this, they adapt heuristic algorithms based on Teitz and Bart [15] for facility location and Gillett and Johnson [6] for demand allocation. In addition, they employ an exact branch-and-bound method to benchmark the performance of the heuristics. The authors apply these methods to several scenarios within a metropolitan region's distribution network, demonstrating that the proposed heuristics yield high-quality solutions with competitive computational efficiency. The study underscores the effectiveness of combining combinatorial optimization techniques with heuristic algorithms in the addressing of complex planning problems in power distribution systems.

On the other hand, Berger et al. [2] address the challenge of locating renewable power generation assets by leveraging extensive climatological data to account for spatio-temporal complementarity. They formulate the problem as a combinatorial optimization task aiming to select a specified number of sites that minimize the occurrence of simultaneous low-electricity production events, relative to a predefined reference production level. The model is closely related to submodular optimization and extends the well-known maximum coverage problem.

The authors propose various deterministic and randomized algorithms, including greedy, local search, and relaxation-based heuristics, as well as combinations thereof. These methods are benchmarked against a state-of-the-art mixed-integer programming solver using a realistic case study inspired by the siting of onshore wind power plants in Europe. The results demonstrate that the proposed algorithms consistently produce better solutions at a fraction of the computational cost. Furthermore, a cross-validation analysis indicates that the model can reliably identify deployment patterns that perform well on previously unseen climatological data, with the exception of an edge case.

10.2.3 Research gap

Despite the advances enabled by linear programming, combinatorial optimization, and heuristic/metaheuristic methods, several limitations remain in their application to electricity system planning. Linear programming approaches, while computationally efficient and capable of producing globally optimal solutions for linear problems, often rely on simplifications such as linearized power flows and deterministic inputs, which can reduce accuracy when modeling real-world, non-linear, and stochastic systems. Combinatorial optimization methods effectively handle discrete decisions such as substation placement or network topology design but face scalability challenges due to their NP-hard nature and frequently provide only near-optimal solutions in large networks. Heuristic and metaheuristic algorithms offer flexibility for complex and non-linear problems but are sensitive to parameter tuning, problem-specific design, and computational time, and often lack theoretical guarantees regarding solution quality. Moreover, most existing techniques predominantly focus on single-objective optimization, neglecting the simultaneous consideration of cost, reliability, emissions, and resilience, and they rarely integrate long-term planning with real-time operational variability. These gaps highlight the need for approaches that can incorporate uncertainty, multi-objective optimization, and scalable, robust solutions, particularly in modern grids with high shares of renewable energy and distributed resources.

Bibliography X

[1] Jonas Allegrini, Kristina Orehounig, Georgios Mavromatidis, Florian Ruesch, Viktor Dorer, and Ralph Evins. "A review of modelling approaches and tools for the simulation of district-scale energy systems". In: *Renewable and Sustainable Energy Reviews* 52 (2015), pp. 1391–1404. doi: 10.1016/j.rser.2015.07.123.

[2] Mathias Berger, David Radu, Antoine Dubois, Hrvoje Pandžić, Yury Dvorkin, Quentin Louveaux, and Damien Ernst. "Siting renewable power generation assets with combinatorial optimisation". In: *Optimization Letters* 15.4 (2021), pp. 877–907. doi: 10.1007/s11590-021-01795-0.

[3] Christopher T. M. Clack, Yuanfu Xie, and Alexander E. MacDonald. "Linear programming techniques for developing an optimal electrical system including high-voltage direct-current transmission and storage". In: *International Journal of Electrical Power & Energy Systems* 68 (2015), pp. 103–114. doi: 10.1016/j.ijepes.2014.12.049.

[4] Yan Ding, Qiaochu Wang, Zhe Tian, Yacong Lyu, Feng Li, Zhe Yan, and Xi Xia. "A graph-theory-based dynamic programming planning method for distributed energy system planning: Campus area as a case study". In: *Applied Energy* 329 (2023), paper no. 120258. doi: 10.1016/j.apenergy.2022.120258.

[5] Tobias Falke, Stefan Krengel, Ann-Kathrin Meinerzhagen, and Armin Schnettler. "Multi-objective optimization and simulation model for the design of distributed energy systems". In: *Applied Energy* 184 (2016), pp. 1508–1516. doi: 10.1016/j.apenergy.2016.03.044.

[6] Billy E. Gillett and Jerry J. Johnson. "Multi-terminal vehicle-dispatch algorithm". In: *Omega* 4.6 (1976), pp. 711–718. doi: 10.1016/0305-0483(76)90097-9.

[7] Chenke He, Jizhong Zhu, Fengji Luo, Yun Liu, Yanjiang Li, and Linying Huang. "A graph theory-based optimal planning method for energy supply networks in an integrated energy system". In: *Sustainable Energy, Grids and Networks* 35 (2023), pp. 101–108. doi: 10.1016/j.segan.2023.101108.

[8] Takayuki Ishizaki, Aranya Chakraborty, and Jun-Ichi Imura. "Graph-theoretic analysis of power systems". In: *Proceedings of the IEEE* 106.5 (2018), pp. 931–952. doi: 10.1109/JPROC.2018.2812298.

[9] Aaron Jaech, Baosen Zhang, Mari Ostendorf, and Daniel S. Kirschen. "Real-time prediction of the duration of distribution system outages". In: *IEEE Transactions on Power Systems* 34.1 (2019), pp. 773–781. doi: 10.1109/TPWRS.2018.2860904.

[10] Pierluigi Mancarella. "MES (multi-energy systems): An overview of concepts and evaluation models". In: *Energy* 65 (2014), pp. 1–17. doi: https://doi.org/10.1016/j.energy.2013.10.041.

[11] Gabriel Rafalski Breda and Mário Mestria. "Optimization methods for planning in electric power grids = Métodos de otimização para planejamento em redes elétricas". In: *Revista Gestão e Secretariado* 14.5 (2023), pp. 8431–8451. doi: 10.7769/gesec.v14i5.2220.

[12] Victor H. Gonzalez-Sanchez, V. Torres-García, Daniel Guillen, and Luis M. Castro. "Graph theory-based fault location Method for transmission systems with renewable energy sources". In: *IEEE Open Access Journal of Power and Energy* 11 (2024), pp. 611–623. doi: 10.1109/oajpe.2024.3507537.

[13] Rama Sireesha, Coppisetty Srinivasa Rao, and Mahesh Vijay Kumar. "Graph theory based transformation of existing distribution network into clusters of multiple micro-grids for reliability enhancement". In: *Materials Today: Proceedings* 47.2 (2021), pp. 120–126. doi: 10.1016/j.matpr.2021.07.067.

[14] Sukono Sukono, Eman Lesmana, Boy Bagja Nugraha, Sudradjat Supian Supian, Jumadil Saputra, and Abdul Talib Bin Bon. "Linear programming for electrical energy generation power plant: An economic optimization approach". In: *Utopía y Praxis Latinoamericana* 25.Esp.2 (2020), pp. 8431–8451. url: https://www.redalyc.org/journal/279/27963185016/27963185016.pdf (visited on 09/10/2025).

[15] Michael B. Teitz and Polly Bart. "Heuristic methods for estimating the generalized vertex median of a weighted graph". In: *Operations Research* 16.5 (1968), pp. 955–961. doi: 10.1287/opre.16.5.955.

[16] Baozhuo Xie, Chao Qi, Hongqi Ben, and Weixiong Yu. "The applications of graph theory in electric network". In: *International Conference on Sensing, Diagnostics, Prognostics, and Control*. 2019, pp. 780–784. doi: 10.1109/SDPC.2019.00148.